Scalars Versus Vectors
Scalar: number with units
Vector: quantity with magnitude and direction
How to get to the library: need to know how far and which way
MEASURING THE SAME DIRECTION IN DIFFERENT WAYS

- 90° North
- 120°
- 360°
- 30° West of North
- 30° Left of +y
- 60° North of West
- 60° Above - x
- 0° East
- -240°
The Components of a Vector
Even though you know how far and in which direction the library is, you may not be able to walk there in a straight line:
The Components of a Vector
Can resolve vector into perpendicular components using a two-dimensional coordinate system:

\[ \begin{align*}
  r &= 1.50 \text{ m} \\
  \theta &= 25.0^\circ \\
  \vec{r} &= \begin{cases} 
    r_x = 1.36 \text{ m} \\
    r_y = 0.634 \text{ m}
  \end{cases}
\end{align*} \]
Length, angle, and components can be calculated from each other using trigonometry:
Signs of vector components:

(a) $A_x > 0$, $A_y < 0$

(b) $A_x < 0$, $A_y < 0$

(c) $A_x < 0$, $A_y > 0$

(d) $A_x > 0$, $A_y > 0$
Adding vectors graphically:
Place the tail of the second at the head of the first. The sum points from the tail of the first to the head of the last.

Adding Vectors Using Components:
1. Find the components of each vector to be added.
2. Add the $x$- and $y$-components separately.
3. Find the resultant vector.
Subtracting Vectors:
The negative of a vector is a vector of the same magnitude pointing in the opposite direction. Here,
\[ \vec{D} = \vec{A} - \vec{B} \]
Unit vectors are dimensionless vectors of unit length.

Multiplying unit vectors by scalars: the multiplier changes the length, and the sign indicates the direction.
Position, Displacement, Velocity, and Acceleration Vectors

Position vector $\vec{r}_f$ points from the origin to the location in question.
The displacement vector $\Delta \vec{r}$ points from the original position to the final position.
The negative of a vector is just another vector of the same magnitude but pointing in the opposite direction. So \( \mathbf{B} \) is the negative of \( -\mathbf{B} \); it has the same length but opposite direction.
The vector $\mathbf{A}$, with its tail at the origin of an $x$, $y$-coordinate system, is shown together with its $x$- and $y$-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle. The analytical relationships among these vectors are summarized below.
The magnitudes of the vector components $A_x$ and $A_y$ can be related to the resultant vector $A$ and the angle $\theta$ with trigonometric identities. Here we see that $A_x = A \cos \theta$ and $A_y = A \sin \theta$. 
The magnitude and direction of the resultant vector can be determined once the horizontal and vertical components $A_x$ and $A_y$ have been determined.
Vectors $\mathbf{A}$ and $\mathbf{B}$ are two legs of a walk, and $\mathbf{R}$ is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$. 

$$\mathbf{A} + \mathbf{B} = \mathbf{R}$$
To add vectors $\mathbf{A}$ and $\mathbf{B}$, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$, $\mathbf{B}_x$ and $\mathbf{B}_y$ shown in the image.
The magnitude of the vectors $A_x$ and $B_x$ add to give the magnitude $R_x$ of the resultant vector in the horizontal direction. Similarly, the magnitudes of the vectors $A_y$ and $B_y$ add to give the magnitude $R_y$ of the resultant vector in the vertical direction.
Vector A has magnitude 53.0 m and direction 20.0° north of the x-axis. Vector B has magnitude 34.0 m and direction 63.0° north of the x-axis. You can use analytical methods to determine the magnitude and direction of \( \mathbf{R} \).
Using analytical methods, we see that the magnitude of $\mathbf{R}$ is 81.2 m and its direction is $36.6^\circ$ north of east.
The subtraction of the two vectors shown in Figure above. The components of \(-B\) are the negatives of the components of \(B\). The method of subtraction is the same as that for addition.
Two-Dimensional Kinematics
If velocity is constant, motion is along a straight line:
Motion in the $x$- and $y$-directions should be solved separately:

<table>
<thead>
<tr>
<th>Position as a function of time</th>
<th>Velocity as a function of time</th>
<th>Velocity as a function of position</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$</td>
<td>$v_x = v_{0x} + a_xt$</td>
<td>$v_x^2 = v_{0x}^2 + 2a_x\Delta x$</td>
</tr>
<tr>
<td>$y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$</td>
<td>$v_y = v_{0y} + a_yt$</td>
<td>$v_y^2 = v_{0y}^2 + 2a_y\Delta y$</td>
</tr>
</tbody>
</table>
Projectile Motion: Basic Equations

Assumptions:

- ignore air resistance
- $g = 9.81 \text{ m/s}^2$, downward
- ignore Earth’s rotation If $y$-axis points upward, acceleration in $x$-direction is zero and acceleration in $y$-direction is $-9.81 \text{ m/s}^2$

The acceleration is independent of the direction of the velocity:
Projectile Motion: Basic Equations

These, then, are the basic equations of projectile motion:

\[ x = x_0 + v_{0x}t \]
\[ y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \]

\[ v_x = v_{0x} \]
\[ v_y = v_{0y} - gt \]

\[ v_x^2 = v_{0x}^2 \]
\[ v_y^2 = v_{0y}^2 - 2g\Delta y \]
Zero Launch Angle
Launch angle: direction of initial velocity with respect to horizontal
Zero Launch Angle

In this case, the initial velocity in the $y$-direction is zero. Here are the equations of motion, with $x_0 = 0$ and $y_0 = h$:

\[ x = v_0 t \]

\[ y = h - \frac{1}{2} gt^2 \]

$v_x = v_0 = \text{constant} \quad v_x^2 = v_0^2 = \text{constant}$

$v_y = -gt \quad v_y^2 = -2g \Delta y$
Zero Launch Angle
This is the trajectory of a projectile launched horizontally:
Eliminating \( t \) and solving for \( y \) as a function of \( x \):

\[
y = h - \frac{1}{2} g \left( \frac{x}{v_0} \right)^2 = h - \left( \frac{g}{2v_0^2} \right) x^2
\]

This has the form \( y = a + bx^2 \), which is the equation of a parabola.

The landing point can be found by setting \( y = 0 \) and solving for \( x \):

\[
x = v_0 \sqrt{\frac{2h}{g}}
\]
General Launch Angle

In general, $v_{0x} = v_0 \cos \theta$ and $v_{0y} = v_0 \sin \theta$. This gives the equations of motion:

\[x = (v_0 \cos \theta) t\]

\[y = (v_0 \sin \theta) t - \frac{1}{2} gt^2\]

\[v_x = v_0 \cos \theta\]

\[v_x^2 = v_0^2 \cos^2 \theta\]

\[v_y = v_0 \sin \theta - gt\]

\[v_y^2 = v_0^2 \sin^2 \theta - 2g \Delta y\]
Snapshots of a trajectory; red dots are at $t = 1\ s$, $t = 2\ s$, and $t = 3\ s$
Projectile Motion: Key Characteristics

Range: the horizontal distance a projectile travels. If the initial and final elevation are the same:

\[ R = \left( \frac{v_0^2}{g} \right) \sin 2\theta \]
The range is a maximum when $\theta = 45^\circ$:

\[ R_{\text{max}} = \frac{v_0^2}{g} \]
Symmetry in projectile motion:
- Components of motion in the $x$- and $y$-directions can be treated independently.
- In projectile motion, the acceleration is $-g$.
- If the launch angle is zero, the initial velocity has only an $x$-component.
- The path followed by a projectile is a parabola.
- The range is the horizontal distance the projectile travels.
(a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes.

(b) The horizontal motion is simple, because $a_x = 0$ and $v_x$ is thus constant.

(c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity.

(d) The $x$- and $y$-motions are recombined to give the total velocity at any given point on the trajectory.
The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.
The trajectory of a rock ejected from the Kilauea volcano.
Trajectories of projectiles on level ground.

(a) The greater the initial speed $v_0$, the greater the range for a given initial angle.

(b) The effect of initial angle $\theta_0$ on the range of a projectile with a given initial speed. Note that the range is the same for $15^\circ$ and $75^\circ$, although the maximum heights of those paths are different.
Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.
A boat trying to head straight across a river will actually move diagonally relative to the shore as shown. Its total velocity (solid arrow) relative to the shore is the sum of its velocity relative to the river plus the velocity of the river relative to the shore.
An airplane heading straight north is instead carried to the west and slowed down by wind. The plane does not move relative to the ground in the direction it points; rather, it moves in the direction of its total velocity (solid arrow).
A boat attempts to travel straight across a river at a speed 0.75 m/s. The current in the river, however, flows at a speed of 1.20 m/s to the right. What is the total displacement of the boat relative to the shore?
An airplane is known to be heading north at 45.0 m/s, though its velocity relative to the ground is 38.0 m/s at an angle west of north. What is the speed and direction of the wind?
Classical relativity. The same motion as viewed by two different observers. An observer on the moving ship sees the binoculars dropped from the top of its mast fall straight down. An observer on shore sees the binoculars take the curved path, moving forward with the ship. Both observers see the binoculars strike the deck at the base of the mast. The initial horizontal velocity is different relative to the two observers.
• The motion of a coin dropped inside an airplane as viewed by two different observers.

(a) An observer in the plane sees the coin fall straight down.

(b) An observer on the ground sees the coin move almost horizontally.
Average velocity vector: \[ \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} \]

So \( \vec{v}_{av} \) is in the same direction as \( \Delta \vec{r} \)
Instantaneous velocity vector is tangent to the path:
Average acceleration vector is in the direction of the change in velocity:

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$
Velocity vector is always in the direction of motion; acceleration vector can point anywhere:
Relative Motion

The speed of the passenger with respect to the ground depends on the relative directions of the passenger’s and train’s speeds:

(a) 1.2 m/s right, 15.0 m/s right

(b) 1.2 m/s left, 15.0 m/s left
\[ \vec{v}_{pg} = \vec{v}_{pt} + \vec{v}_{tg} \]

This also works in two dimensions:
Summary

• Scalar: number, with appropriate units
• Vector: quantity with magnitude and direction
  • Vector components: $A_x = A \cos \theta$, $B_y = B \sin \theta$
  • Magnitude: $A = (A_x^2 + A_y^2)^{1/2}$
  • Direction: $\theta = \tan^{-1}(A_y / A_x)$
  • Graphical vector addition: Place tail of second at head of first; sum points from tail of first to head of last

• Component method: add components of individual vectors, then find magnitude and direction
• Unit vectors are dimensionless and of unit length
• Position vector points from origin to location
• Displacement vector points from original position to final position
• Velocity vector points in direction of motion
• Acceleration vector points in direction of change of motion
• Relative motion: $\vec{v}_{13} = \vec{v}_{12} + \vec{v}_{23}$