CEEN 3304 Concrete Design

One-way slabs

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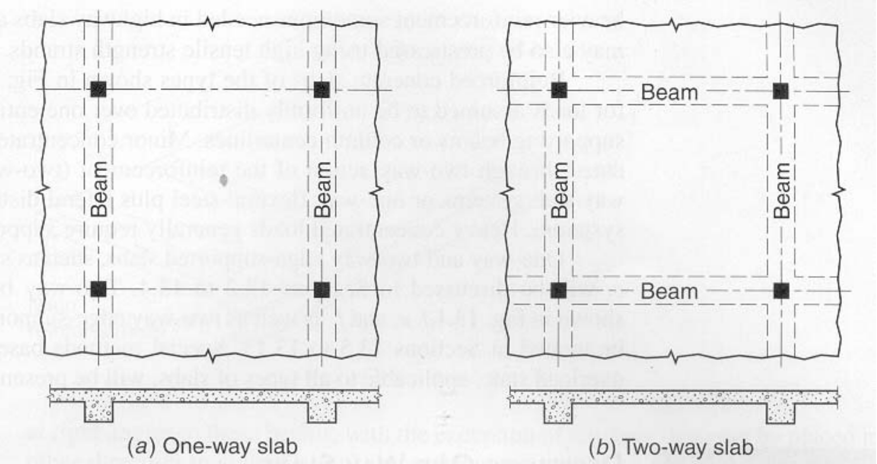
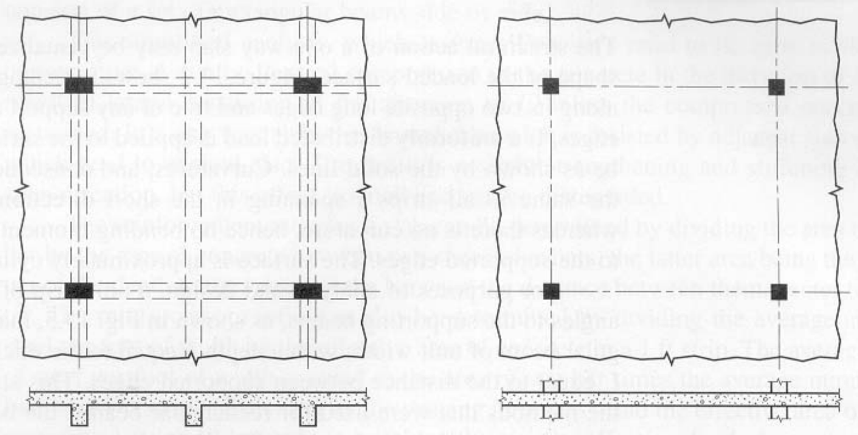
Slab types



(*a*) One-way slab

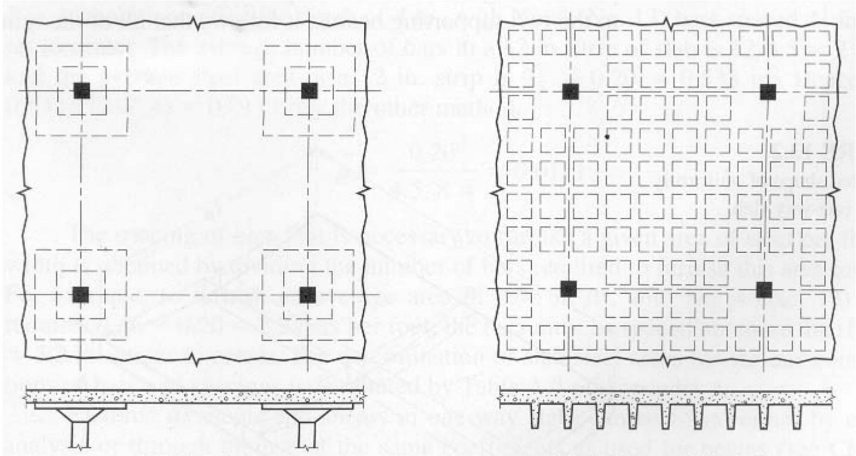
(*b*) Two-way slab

Slab types



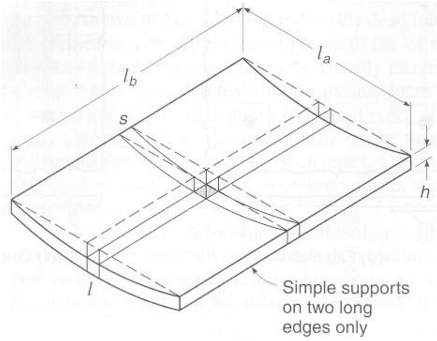
(*c*) One-way slab (*d*) Flat plate

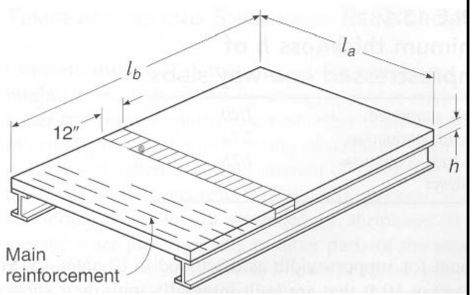
Slab types



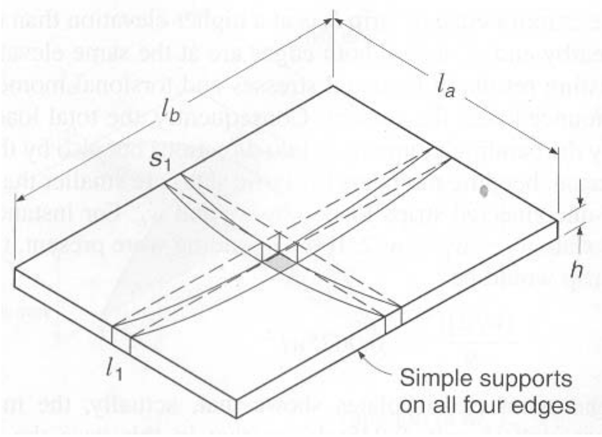
(*e*) Flat slab (*f*) Grid or waffle slab

One-way slab behavior





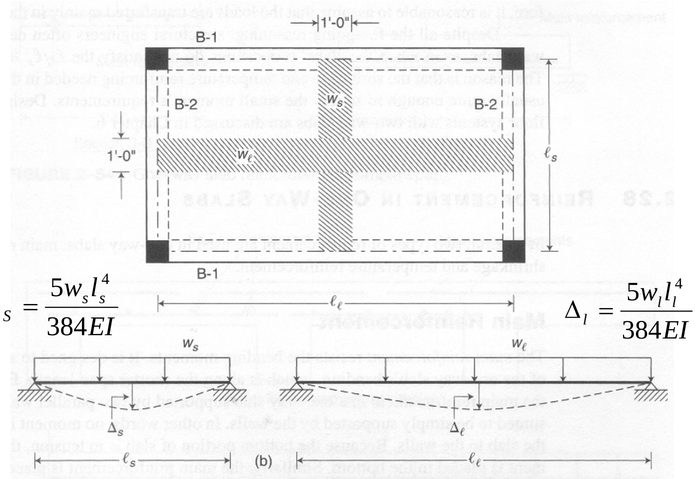
Two-way slab deformation



Two-way slab deformation

- **If l_l/l_s = 2 w_s = 16 w_l**

$$\frac{w_s}{w_l} = \left(\frac{l_l}{l_s}\right)^4$$

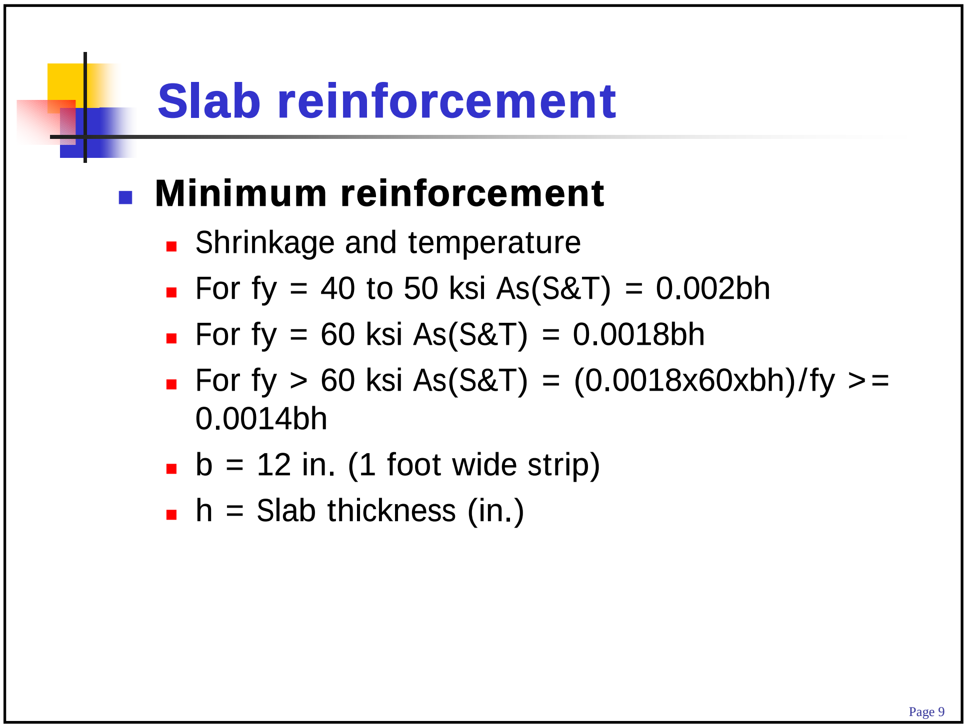


$$\Delta_S = \frac{5 w_s l_s^4}{384EI}$$

$$\Delta_l = \frac{5 w_l l_l^4}{384EI}$$

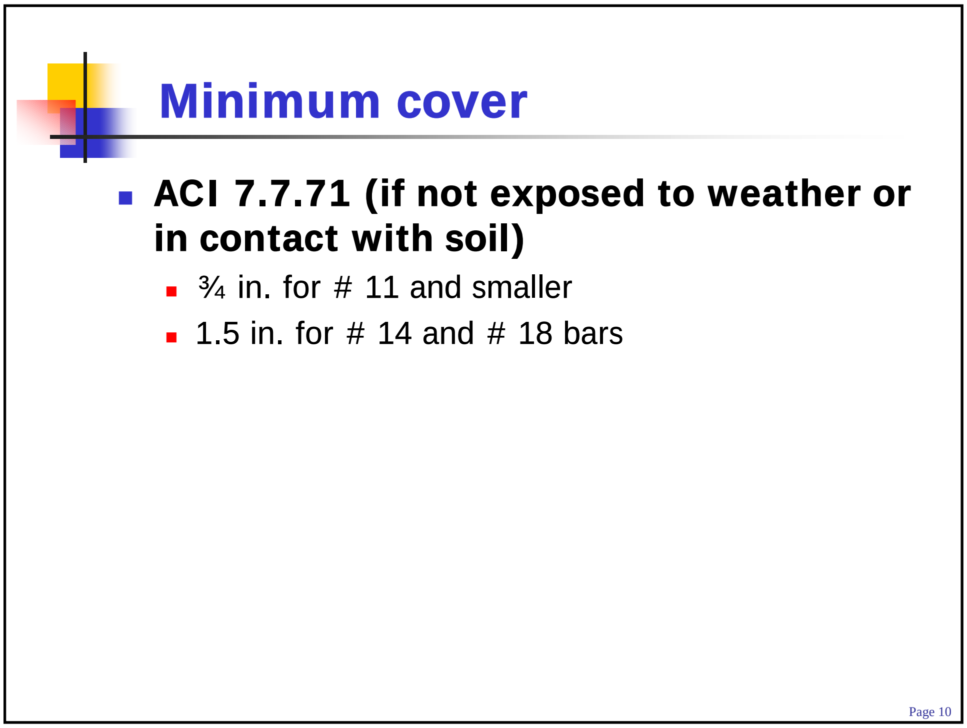
Slab reinforcement

- **Main reinforcement**
  - Used to resist moment
- **Shrinkage and temperature reinforcement**
  - Used to prevent cracking due to contraction of the concrete resulting from shrinkage and temperature changes



Slab reinforcement

- **Minimum reinforcement**
  - Shrinkage and temperature
  - For fy = 40 to 50 ksi As(S&T) = 0.002bh
  - For fy = 60 ksi As(S&T) = 0.0018bh
  - For fy > 60 ksi As(S&T) = (0.0018x60xbh)/fy >= 0.0014bh
  - b = 12 in. (1 foot wide strip)
  - h = Slab thickness (in.)



Minimum cover

- **ACI 7.7.71 (if not exposed to weather or in contact with soil)**
  - ¾ in. for # 11 and smaller
  - 1.5 in. for # 14 and # 18 bars

Maximum spacing

- **Main reinforcement**
  - ACI 10.5.4
  - S < smaller of (3 h or 18 in.)
  - ACI 10.6.4 $$s = 15\left(\frac{40000}{f_s}\right) - 2.5c_c \le 12\left(\frac{40000}{f_s}\right)$$
  - Using min cover of ¾ in. and fs = 2/3fy=2/3(60000) = 40,000 psi
  - S = 12 in.
- **Shrinkage and Temp. reinforcement**
  - ACI 7.12.2.2
  - Smax (S&T) = min(5h or 18 in.)

Example

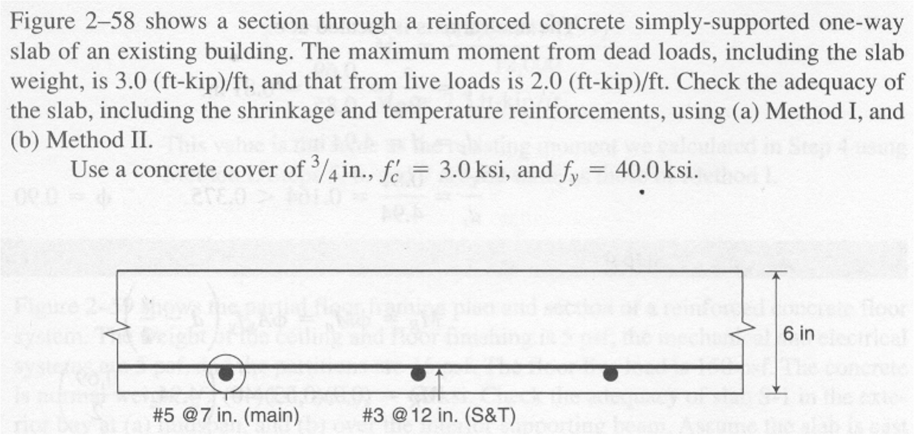
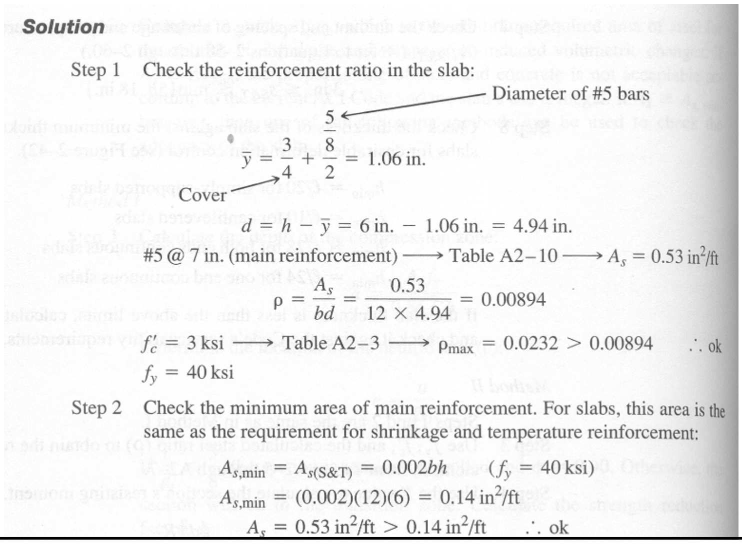


Figure 2–58 shows a section through a reinforced concrete simply-supported one-way slab of an existing building. The maximum moment from dead loads, including the slab weight, is 3.0 (ft-kip)/ft, and that from live loads is 2.0 (ft-kip)/ft. Check the adequacy of the slab, including the shrinkage and temperature reinforcements, using (a) Method I, and (b) Method II.

Use a concrete cover of $^3/_4$ in., $f_c' = 3.0$ ksi, and $f_y = 40.0$ ksi.

Example



Solution

Step 1 Check the reinforcement ratio in the slab:

Diameter of #5 bars

$$\bar{y} = \frac{3}{4} + \frac{\frac{5}{8}}{2} = 1.06\text{ in.}$$

Cover

$$d = h - \bar{y} = 6\text{ in.} - 1.06\text{ in.} = 4.94\text{ in.}$$

$$\#5 \text{ @ } 7\text{ in. (main reinforcement)} \longrightarrow \text{Table A2–10} \longrightarrow A_s = 0.53\text{ in}^2/\text{ft}$$

$$\rho = \frac{A_s}{bd} = \frac{0.53}{12 \times 4.94} = 0.00894$$

$$\begin{array}{l} f'_c = 3\text{ ksi} \longrightarrow \text{Table A2–3} \longrightarrow \rho_{\max} = 0.0232 > 0.00894 \qquad \therefore \text{ok} \\ f_y = 40\text{ ksi} \end{array}$$

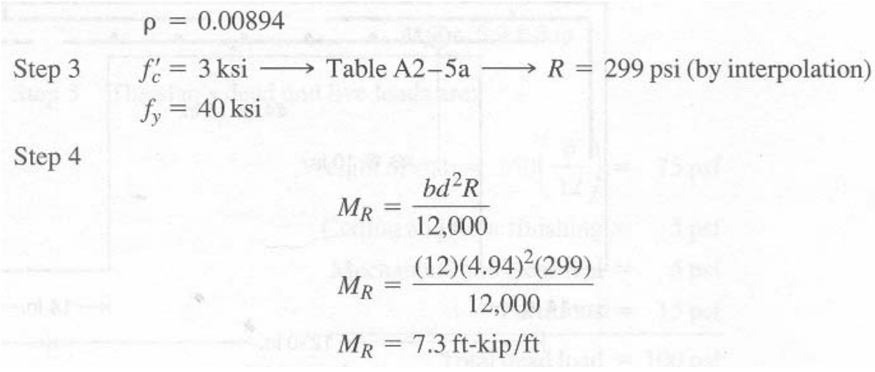
Step 2 Check the minimum area of main reinforcement. For slabs, this area is the same as the requirement for shrinkage and temperature reinforcement:

$$A_{s,\min} = A_{s(\text{S\&T})} = 0.002bh \qquad (f_y = 40\text{ ksi})$$

$$A_{s,\min} = (0.002)(12)(6) = 0.14\text{ in}^2/\text{ft}$$

$$A_s = 0.53\text{ in}^2/\text{ft} > 0.14\text{ in}^2/\text{ft} \qquad \therefore \text{ok}$$

Example



Step 3

$$\begin{array}{l} \rho = 0.00894 \\ f'_c = 3\text{ ksi} \longrightarrow \text{Table A2–5a} \longrightarrow R = 299\text{ psi (by interpolation)} \\ f_y = 40\text{ ksi} \end{array}$$

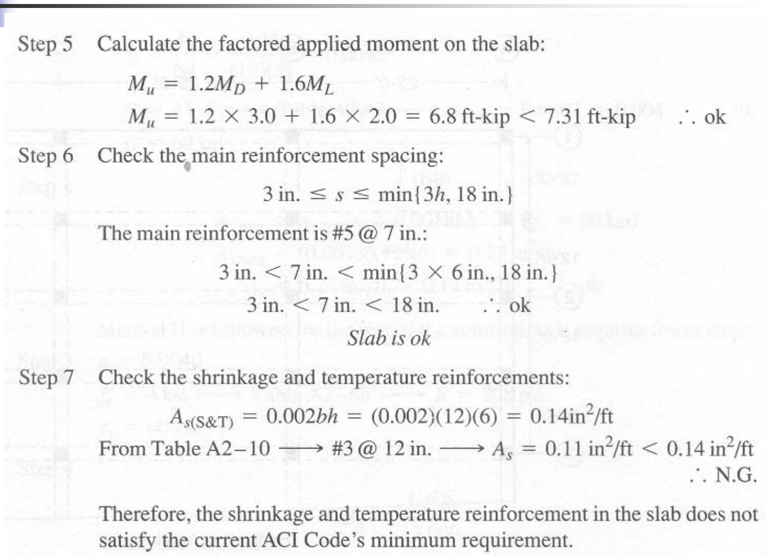
Step 4

$$M_R = \frac{bd^2R}{12{,}000}$$

$$M_R = \frac{(12)(4.94)^2(299)}{12{,}000}$$

$$M_R = 7.3\text{ ft-kip/ft}$$

Example



Step 5 Calculate the factored applied moment on the slab:

$$M_u = 1.2M_D + 1.6M_L$$

$$M_u = 1.2 \times 3.0 + 1.6 \times 2.0 = 6.8 \text{ ft-kip} < 7.31 \text{ ft-kip} \qquad \therefore \text{ ok}$$

Step 6 Check the main reinforcement spacing:

$$3 \text{ in.} \leq s \leq \min\{3h, 18 \text{ in.}\}$$

The main reinforcement is #5 @ 7 in.:

$$3 \text{ in.} < 7 \text{ in.} < \min\{3 \times 6 \text{ in.}, 18 \text{ in.}\}$$

$$3 \text{ in.} < 7 \text{ in.} < 18 \text{ in.} \qquad \therefore \text{ ok}$$

Slab is ok

Step 7 Check the shrinkage and temperature reinforcements:

$$A_{s(\text{S\&T})} = 0.002bh = (0.002)(12)(6) = 0.14\text{in}^2/\text{ft}$$

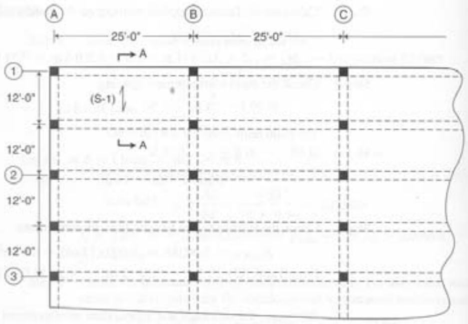
From Table A2–10 $\longrightarrow$ #3 @ 12 in. $\longrightarrow A_s = 0.11 \text{ in}^2/\text{ft} < 0.14 \text{ in}^2/\text{ft}$

$\therefore$ N.G.

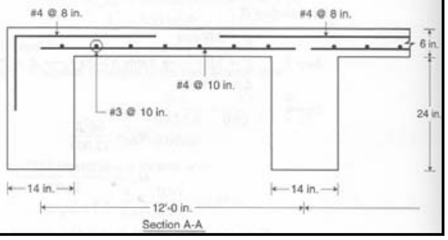
Therefore, the shrinkage and temperature reinforcement in the slab does not satisfy the current ACI Code's minimum requirement.

Design example

- **Determine the reinforcement at midspan and at the supports**

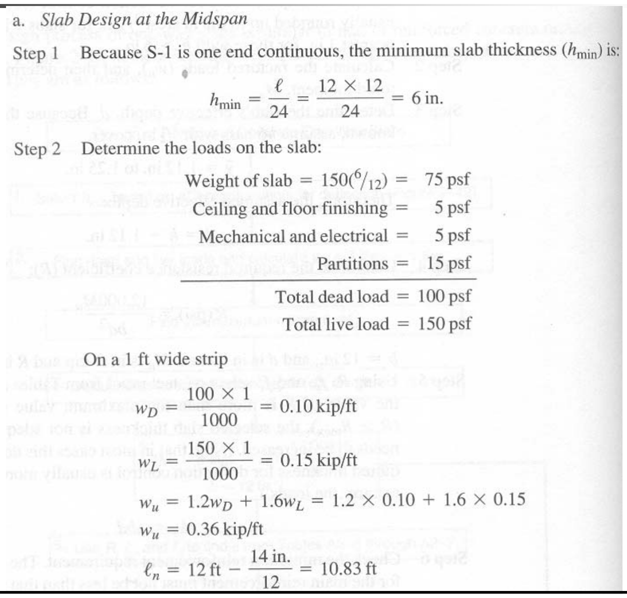


Partial floor framing plan



Section A-A

Design example



a. *Slab Design at the Midspan*

Step 1 Because S-1 is one end continuous, the minimum slab thickness ($h_{\min}$) is:

$$h_{\min} = \frac{\ell}{24} = \frac{12 \times 12}{24} = 6 \text{ in.}$$

Step 2 Determine the loads on the slab:

Weight of slab = $150(^6/_{12})$ = 75 psf
Ceiling and floor finishing = 5 psf
Mechanical and electrical = 5 psf
Partitions = 15 psf
Total dead load = 100 psf
Total live load = 150 psf

On a 1 ft wide strip

$$w_D = \frac{100 \times 1}{1000} = 0.10 \text{ kip/ft}$$

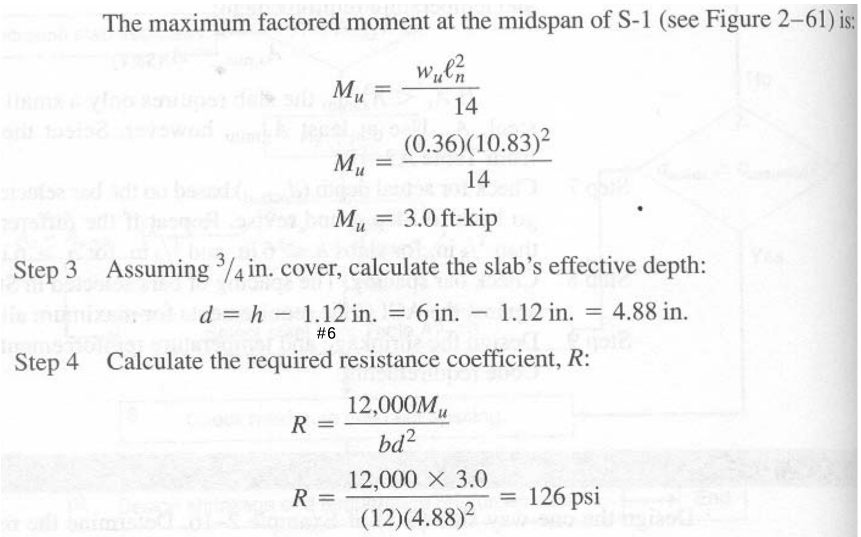
$$w_L = \frac{150 \times 1}{1000} = 0.15 \text{ kip/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2 \times 0.10 + 1.6 \times 0.15$$

$$w_u = 0.36 \text{ kip/ft}$$

$$\ell_n = 12 \text{ ft} - \frac{14 \text{ in.}}{12} = 10.83 \text{ ft}$$

Design example



The maximum factored moment at the midspan of S-1 (see Figure 2–61) is:

$$M_u = \frac{w_u \ell_n^2}{14}$$

$$M_u = \frac{(0.36)(10.83)^2}{14}$$

$$M_u = 3.0 \text{ ft-kip}$$

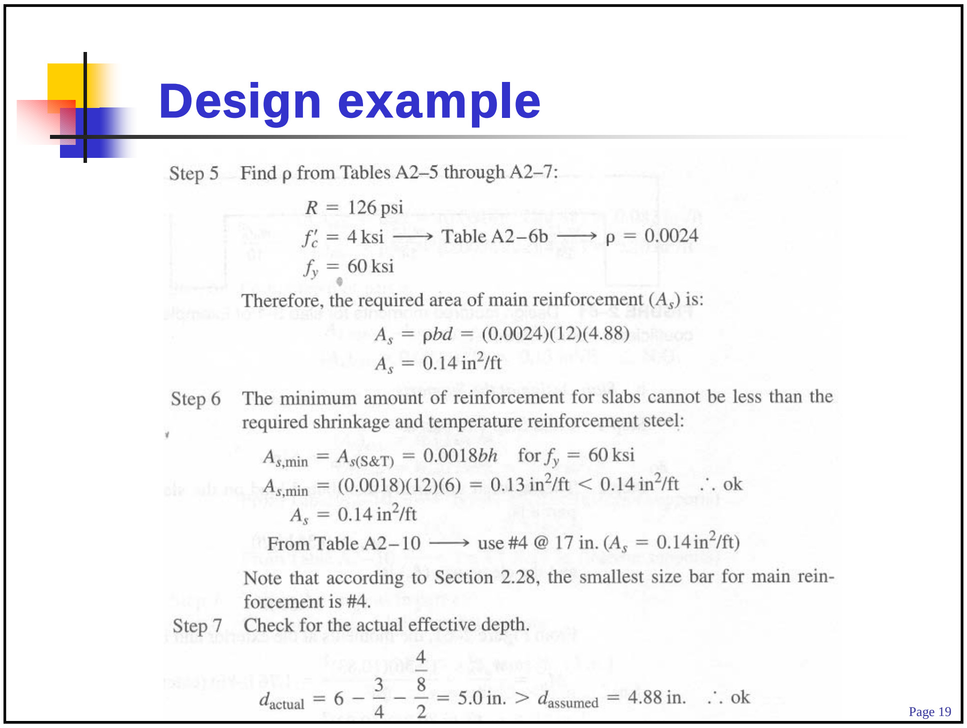
Step 3 Assuming $^3/_4$ in. cover, calculate the slab's effective depth:

$$d = h - \underset{\#6}{1.12 \text{ in.}} = 6 \text{ in.} - 1.12 \text{ in.} = 4.88 \text{ in.}$$

Step 4 Calculate the required resistance coefficient, R:

$$R = \frac{12{,}000 M_u}{bd^2}$$

$$R = \frac{12{,}000 \times 3.0}{(12)(4.88)^2} = 126 \text{ psi}$$



Design example

Step 5 Find ρ from Tables A2–5 through A2–7:

$$R = 126\text{ psi}$$
$$f'_c = 4\text{ ksi} \longrightarrow \text{Table A2–6b} \longrightarrow \rho = 0.0024$$
$$f_y = 60\text{ ksi}$$

Therefore, the required area of main reinforcement (A_s) is:

$$A_s = \rho bd = (0.0024)(12)(4.88)$$
$$A_s = 0.14\text{ in}^2/\text{ft}$$

Step 6 The minimum amount of reinforcement for slabs cannot be less than the required shrinkage and temperature reinforcement steel:

$$A_{s,\min} = A_{s(\text{S\&T})} = 0.0018bh \quad \text{for } f_y = 60\text{ ksi}$$
$$A_{s,\min} = (0.0018)(12)(6) = 0.13\text{ in}^2/\text{ft} < 0.14\text{ in}^2/\text{ft} \quad \therefore \text{ ok}$$
$$A_s = 0.14\text{ in}^2/\text{ft}$$
$$\text{From Table A2–10} \longrightarrow \text{use \#4 @ 17 in. } (A_s = 0.14\text{in}^2/\text{ft})$$

Note that according to Section 2.28, the smallest size bar for main reinforcement is #4.

Step 7 Check for the actual effective depth.

$$d_{\text{actual}} = 6 - \frac{3}{4} - \frac{\frac{4}{8}}{2} = 5.0\text{ in.} > d_{\text{assumed}} = 4.88\text{ in.} \quad \therefore \text{ ok}$$

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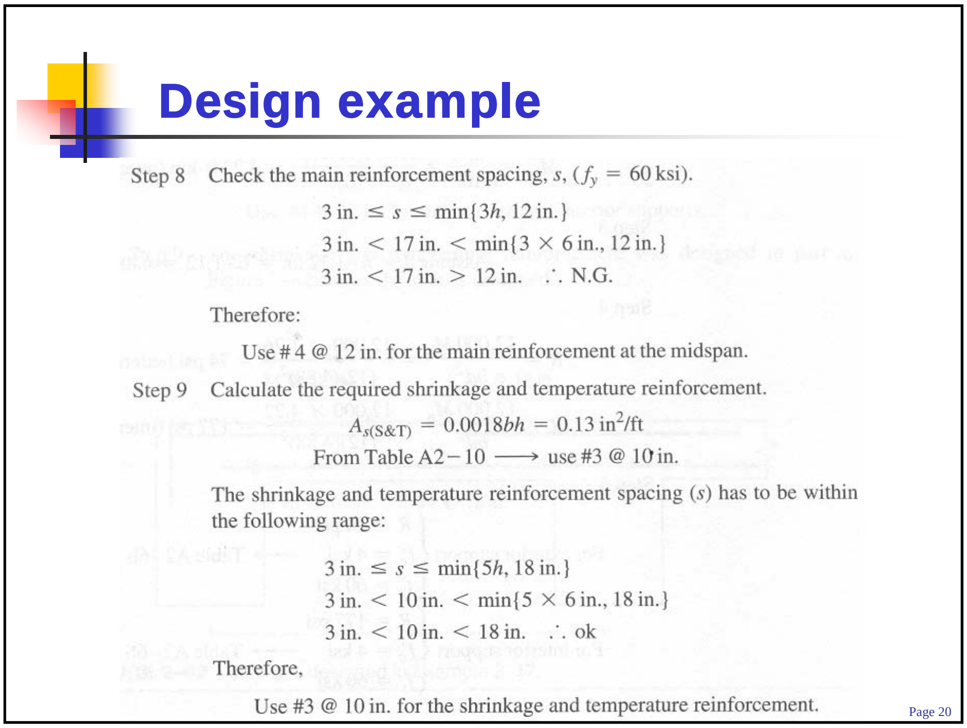

Design example

Step 8 Check the main reinforcement spacing, s, ($f_y = 60$ ksi).

$$3\text{ in.} \le s \le \min\{3h, 12\text{ in.}\}$$
$$3\text{ in.} < 17\text{ in.} < \min\{3 \times 6\text{ in.}, 12\text{ in.}\}$$
$$3\text{ in.} < 17\text{ in.} > 12\text{ in.} \quad \therefore \text{ N.G.}$$

Therefore:

Use # 4 @ 12 in. for the main reinforcement at the midspan.

Step 9 Calculate the required shrinkage and temperature reinforcement.

$$A_{s(\text{S\&T})} = 0.0018bh = 0.13\text{ in}^2/\text{ft}$$
$$\text{From Table A2} - 10 \longrightarrow \text{use \#3 @ 10 in.}$$

The shrinkage and temperature reinforcement spacing (s) has to be within the following range:

$$3\text{ in.} \le s \le \min\{5h, 18\text{ in.}\}$$
$$3\text{ in.} < 10\text{ in.} < \min\{5 \times 6\text{ in.}, 18\text{ in.}\}$$
$$3\text{ in.} < 10\text{ in.} < 18\text{ in.} \quad \therefore \text{ ok}$$

Therefore,

Use #3 @ 10 in. for the shrinkage and temperature reinforcement.

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Design example

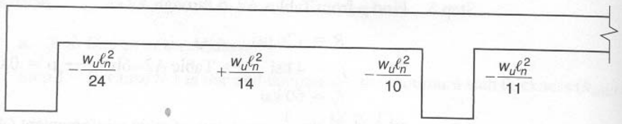

FIGURE 2–61 Design factored moments for slab S-1 of Example 2–17 using ACI Code coefficients from Table A2–1.

b. *Slab Design at the Supports*

Step 1 From Step 1 of part a:

$$h_{\min} = 6 \text{ in.}$$

Step 2 The factored uniformly distributed load on the slab (w_u) from Step 2 of part a is:

$$w_u = 0.36 \text{ kip/ft}$$

and the clear span (ℓ_n) is:

$$\ell_n = 10.83 \text{ ft}$$

From Figure 2–61, the moments at the exterior and interior supports are:

$$M_u^- = \frac{w_u \ell_n^2}{24} = \frac{(0.36)(10.83)^2}{24} = 1.76 \text{ ft-kip (exterior support)}$$

$$M_u^- = \frac{w_u \ell_n^2}{10} = \frac{(0.36)(10.83)^2}{10} = 4.22 \text{ ft-kip (interior support)}$$

Design example

Step 3

$$\text{Assume } d = h - 1.12 \text{ in.} = 6 - 1.12 = 4.88 \text{ in.}$$

Step 4

$$R = \frac{12{,}000\, M_u}{bd^2} = \frac{12{,}000 \times 1.76}{(12)(4.88)^2} = 74 \text{ psi (exterior support)}$$

$$R = \frac{12{,}000\, M_u}{bd^2} = \frac{12{,}000 \times 4.22}{(12)(4.88)^2} = 177 \text{ psi (interior support)}$$

Step 5

$$\text{For exterior support} \begin{cases} R = 74 \text{ psi} \\ f'_c = 4 \text{ ksi} \\ f_y = 60 \text{ ksi} \end{cases} \longrightarrow \text{Table A2–6b} \longrightarrow \rho_{\text{ext.}} = 0.0014$$

$$\text{For interior support} \begin{cases} R = 177 \text{ psi} \\ f'_c = 4 \text{ ksi} \\ f_y = 60 \text{ ksi} \end{cases} \longrightarrow \text{Table A2–6b} \longrightarrow \rho_{\text{int.}} = 0.0034$$

Design example

Therefore:

$$(A_s)_{ext.} = \rho bd = (0.0014)(12)(4.88) = 0.082\ \text{in}^2/\text{ft}$$
$$(A_s)_{int.} = \rho bd = (0.0034)(12)(4.88) = 0.20\ \text{in}^2/\text{ft}$$

Step 6 From Step 6 of part a:

$$A_{s,\min} = A_{s(\text{S\&T})} = 0.13\ \text{in}^2/\text{ft}$$
$$(A_s)_{ext} = 0.082\ \text{in}^2/\text{ft} < 0.13\ \text{in}^2/\text{ft} \quad \therefore \text{N.G.}$$

Therefore, use

$$(A_s)_{ext.} = 0.13\ \text{in}^2/\text{ft}$$
$$(A_s)_{int.} = 0.20\ \text{in}^2/\text{ft} > 0.13\ \text{in}^2/\text{ft} \quad \therefore \text{ok}$$

From Table A2–10 ⟶ Try #4 @ 12 in. (exterior supports)

$$(A_s)_{int.} = 0.20\ \text{in}^2/\text{ft}$$

From Table A2–10 ⟶ Try #4 @ 12 in. (interior supports)

Design example

Step 7 This is the same as in part a.

Step 8 Check the main reinforcement spacing:

$$3\ \text{in.} \leq s \leq \min\{3h, 12\ \text{in.}\}$$
$$3\ \text{in.} \leq s \leq \min\{3 \times 6\ \text{in.}, 12\ \text{in.}\}$$
$$3\ \text{in.} \leq s \leq 12\ \text{in.}$$
$$s_{int.} = s_{ext.} = 12\ \text{in.} = 12\ \text{in.} \quad \therefore \text{ok}$$

∴ Use #4 @ 12 in. for the exterior and interior supports.

Step 9 The shrinkage and temperature reinforcement was designed in part a. Figure 2–62 shows the slab as designed.