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# Internal Rate of Return

## (Multiple Rates of Return Problem)

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Lecture No. 20  
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## Return on Invested Capital

- **Definition 3:** Return on invested capital is defined as the interest rate earned on the *unrecovered project balance* of an investment project. It is commonly known as internal rate of return (IRR).
  - **Example:** A company invests \$10,000 in a computer and results in equivalent annual labor savings of \$4,021 over 3 years. The company is said to earn a *return of 10%* on its investment of \$10,000.
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## Project Balance Calculation:

	<u>0</u>	<u>1</u>	<u>2</u>	<u>3</u>
Beginning project balance		-\$10,000	-\$6,979	-\$3,656
Return on invested capital		-\$1,000	-\$697	-\$365
Payment received	-\$10,000	+\$4,021	+\$4,021	+\$4,021
Ending project balance	-\$10,000	-\$6,979	-\$3,656	0

The firm earns a 10% rate of return on funds that remain **internally invested in the project**. Since the return is **internal** to the project, we call it **internal rate of return**.

## Multiple Rates of Return

- If a project has more than one rate of return, how would you make an accept/reject decision?

## Investment Classification

### Simple Investment

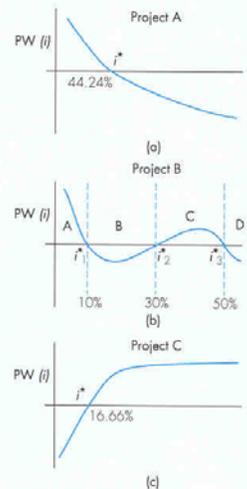
- **Def:** Initial cash flows are negative, and only one sign change occurs in the net cash flows series.
- **Example:** -\$100, \$250, \$300 (-, +, +)
- **ROR:** A unique ROR

### Nonsimple Investment

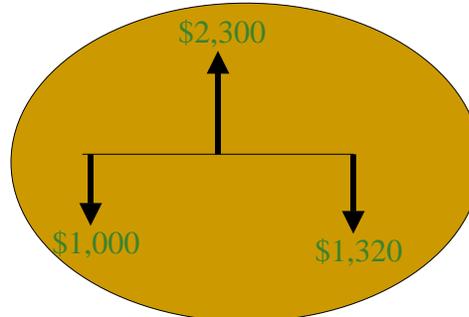
- **Def:** Initial cash flows are negative, but more than one sign changes in the remaining cash flow series.
- **Example:** -\$100, \$300, -\$120 (-, +, -)
- **ROR:** A possibility of multiple RORs

Period (N)	Project A	Project B	Project C
0	-\$1,000	-\$1,000	+\$1,000
1	-500	3,900	-450
2	800	-5,030	-450
3	1,500	2,145	-450
4	2,000		

Project A is a simple investment.  
 Project B is a nonsimple investment.  
 Project C is a simple borrowing.



Example 7.6 Multiple Rates of Return Problem



- Find the rate(s) of return:

$$PW(i) = -\$1,000 + \frac{\$2,300}{1+i} - \frac{\$1,320}{(1+i)^2}$$
$$= 0$$

Let  $x = \frac{1}{1+i}$ . Then,

$$PW(i) = -\$1,000 + \frac{\$2,300}{(1+i)} - \frac{\$1,320}{(1+i)^2}$$
$$= -\$1,000 + \$2,300x - \$1,320x^2$$
$$= 0$$

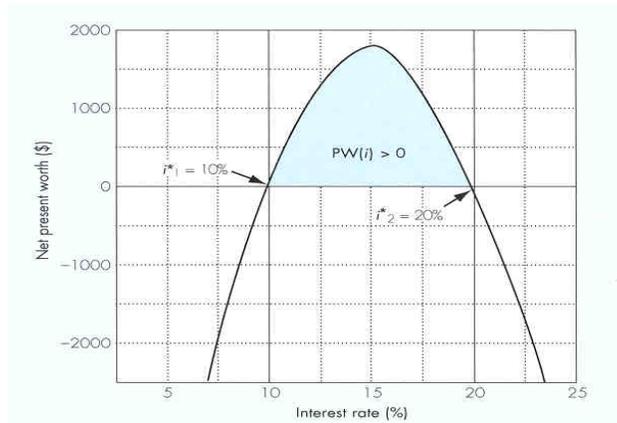
Solving for  $x$  yields,

$$x = 10/11 \text{ or } x = 10/12$$

Solving for  $i$  yields

$$i = 10\% \text{ or } 20\%$$

## PW Plot for a Nonsimple Investment with Multiple Rates of Return



## Project Balance Calculation

$$i^* = 20\%$$

	$n = 0$	$n = 1$	$n = 2$
Beg. Balance		-\$1,000	+\$1,100
Interest		-\$200	+\$220
Payment	-\$1,000	+\$2,300	-\$1,320
Ending Balance	-\$1,000	+\$1,100	\$0

Cash borrowed (released) from the project is assumed to earn the same interest rate through external investment as money that remains internally invested.

**Conceptual Issue:** Can the firm be able to invest the money released from the project at 20% externally in Period 1?

- If the firm's MARR is exactly 20%, the answer is “yes”, because it represents the rate at which the firm can always invest the money in its investment pool. Then, the 20% is also true IRR for the project.
- Suppose the firm's MARR is 15% instead of 20%. The assumption used in calculating  $i^*$  is no longer valid. In order to calculate  $i^*$ , we assumed that all cash released from the project can be invested at the  $i^*$  instead of MARR.
- **Conclusion:** Neither 10% nor 20% is a true IRR.

### How to Proceed:

- If you encounter multiple rates of return, abandon the IRR analysis and use the PW criterion.

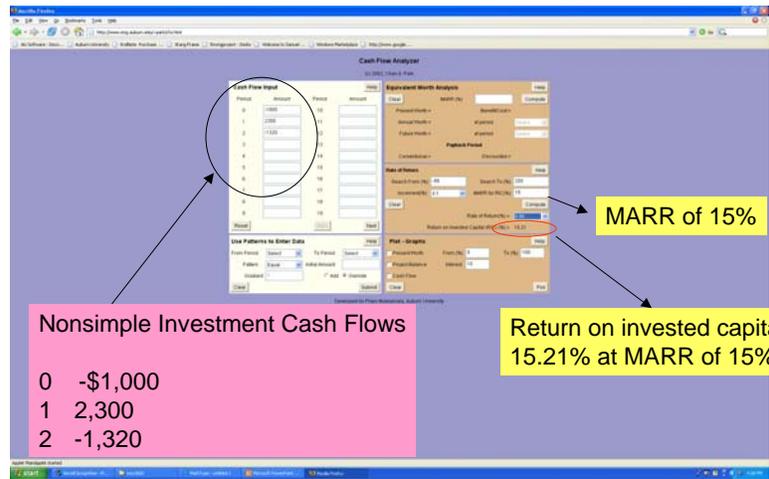
If NPW criterion is used at MARR = 15%

$$\begin{aligned} \text{PW}(15\%) &= -\$1,000 + \$2,300 (P/F, 15\%, 1) \\ &\quad - \$1,320 (P/F, 15\%, 2) \\ &= \$1.89 > 0 \end{aligned}$$

Accept the investment

- If you want to find the true rate of return (or return on invested capital) to the project, follow the procedure outlined in Appendix 7A, or use the Cash Flow Analyzer.

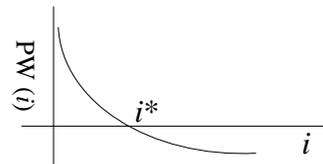
## Finding the True ROR for Nonsimple Projects with Cash Flow Analyzer



## Decision Rules for Nonsimple Investment

- A possibility of multiple RORs.
- If  $PW(i)$  plot looks like this, then,  $IRR = ROR$ .

If  $IRR > MARR$ , Accept



- If  $PW(i)$  plot looks like this, Then,  $IRR \neq ROR (i^*)$ .
  - Find the true IRR by using the procedures in Appendix A or,
  - Abandon the IRR method and use the PW method.

