NONLINEAR MODELING OF FIBER-REINFORCED ELASTOMERS

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ABSTRACT

Accurately predicting the response of fiber-reinforced elastomer or flexible composite structures can be improved by the addition of material, geometric and fiber-rotation nonlinear models to classical laminated plate theory. Material nonlinearity is included in the form of nonlinear orthotropic material properties as functions of extensional strain. Nonlinear properties were obtained from the experimental results of fiber-reinforced elastomeric (FRE) angle-ply specimens at 0°, 45°, and 90° discussed in Reference [xx]. Axial stiffness and Poisson’s ratio are considered constant. Geometric nonlinearity is removed from the transverse and shear stiffnesses. A six-coefficient Ogden model was chosen to represent the nonlinear stiffnesses. Geometric nonlinearity is included through the addition of nonlinear extensional terms from the Lagrangian strain tensor. The nonlinear strain-displacement relations and the nonlinear material models were added to the code of a pre-existing composites analysis software package. The improved analytical tool will aid in understanding the behavior of FRE materials and will enable their use in stiffness- and deformation-tailored smart structures. Because fiber rotation (fiber re-orientation) is a function of geometry and boundary conditions, it is included in a simple model of a “rubber muscle.”

Predicting the response of fiber-reinforced elastomer or flexible composite structures is improved by the addition of material, geometric and fiber-rotation nonlinear models to classical laminated plate theory. FRE stress-strain responses predicted by the improved nonlinear laminated plate model are compared with measured stress-strain responses of balanced angle-ply specimens with off-axis angles ranging from 0° to 90° in 15° increments. Correlation between predicted and experimental results typically ranged from good to excellent.
The response of a “rubber muscle” actuator is also predicted. The rubber muscle model indicates that fiber re-orientation is a function of initial fiber angle and material type, and that very high initial contractive forces are possible.

INTRODUCTION

Increased development of components fabricated from fiber-reinforced elastomers or flexible composites will be limited without the ability to accurately predict the response of such materials. The analysis of traditional composites uses classical lamination theory. Classical lamination theory assumes that strains are small and that orthotropic material properties are linear, however, fiber-reinforced elastomers can experience large strains and typically exhibit nonlinear stress-strain characteristics. Elastomers (rubber) can deform up to 800% and usually have very nonlinear stress-strain curves. Rubber can increase in stiffness when stretched (e.g., silicone), or can decrease in stiffness (e.g., urethane). The deformation of a FRE laminate can change significantly with fiber orientation due to extreme differences in stiffness between matrix and fiber. The difference in matrix and fiber stiffness for typical “stiff” composites is approximately one to two orders of magnitude. The difference in matrix and fiber stiffness for fiber-reinforced elastomeric composites is approximately five orders of magnitude. Laminate Poisson’s ratios, which affect transverse deflection, and are functions of layer stiffnesses, can vary considerably with fiber angle. Because of these unusual characteristics, successful use of FRE materials depends on accurate nonlinear material properties, and the exploitation of those properties in the prediction of FRE response mechanisms.

CONTRIBUTIONS TO THE STATE OF THE ART

The combination of elastomer (rubber) and directional reinforcement into materials such as fiber-reinforced elastomers or elastomer composites is not new, but has primarily used in tires, belting, or impregnated fabrics. The fabrics are constructed such that significant out-of-plane bending is possible, but inplane shear or elongation is restricted. The current work considers FRE laminates where inplane shear and extension are not restricted.

Due to the cording (twisted fibers) in tires and high concentrations of fillers in the rubber, elongation of the reinforced area in a tire is typically under 10%. Relatively little work has been published on the prediction of FRE responses at higher elongations of 20% to 200%. Because of
the smaller strains in cord-rubber composites, linear strain-displacement relations and simple rubber material models, such as the Mooney-Rivlin model are assumed [1,2]. Bert [3] has developed models for cord-rubber composites with different properties in tension and compression. Clark [4] used a bi-linear model of the transverse and shear stiffnesses, with strains up to 10%. His model does not always predict stiffness properly and is directed primarily towards cord-rubber applications. Woo [5] has conducted extensive characterizations of human and animal ligaments and has developed viscoelastic strain models that describe the response of ligaments very well. Woo’s viscoelastic models are not incorporated into the current work but would be useful for future bio-mechanical applications. Chou and Luo [6-8] have conducted perhaps the most comprehensive work on the nonlinear elastic behavior of flexible composites. Their work deals primarily with wavy fibers in an elastomeric matrix. Geometric nonlinearity is introduced through the “straightening” of the wavy fibers as the specimen is elongated. Material nonlinearity is incorporated through use of a third order polynomial material model. Predicted results were compared with test results from specimens with fiber volume fractions on the order of one or two percent. Total elongation of their specimens was approximately twenty percent. In a work completed after the current nonlinear model was created, Derstine [9] combines the fiber and elastomer using a micro-mechanics model, a process science model to determine the local geometry of a 3-D braid, and a stiffness averaging routine for calculating the local stiffness of the material. Fiber re-orientation is calculated. The final stress-strain is used as input in a nonlinear finite element model. Total strain, for the results presented, was under ten percent.

**Current Contributions**

Contributions of the current work include the use of an improved material model, the use of geometric nonlinearity, clarification of the differences between nonlinear strain-displacement and fiber-reorientation, and the prediction of stress-strain responses up to and beyond 200%. Separation of the contributions of nonlinear material properties, geometrically nonlinear strain-displacement relations and nonlinearity due to fiber re-orientation is critical for the development of an accurate model. Modeling of material nonlinearity is improved by use of the extremely accurate Ogden model [10]. Geometric nonlinearity is removed from nonlinear orthotropic properties. This step is necessary because the instruments used to measure strain considered the measured strain to be small and linear. Fiber re-orientation is a function of specimen geometry, rather than
an inherent lamina property. Fiber re-orientation is included in a model of a simple “rubber muscle.” Efforts were made to keep the computer model as simple as possible while retaining accuracy. Results from testing of the constituent rubbers support the conclusion that viscoelastic relations need not be included if quasi-static conditions are imposed, hence viscoelastic relations are not part of the current model.

NONLINEAR MODELING OF FIBER-REINFORCED ELASTOMERS

The nonlinear model is based on classical laminated plate theory with the addition of nonlinear material properties and geometric strain-displacement nonlinearity. An overview of classical laminated plate theory is presented in order to understand the significance of the nonlinear additions. The use of nonlinear material properties requires several supplemental steps and formulae. The actual addition of the nonlinear strain-displacement relations is quite simple but, when coupled with the nonlinear properties, changes the method of solution from closed form to a step or iterative form.

The inclusion of fiber re-orientation (rotation) is geometry and boundary-specific. For example an axi-symmetric FRE cylinder with an off-axis angle of 30 degrees will experience more fiber rotation than a thin flat rectangular angle-ply specimen with the same off-axis angle. As the tube is elongated the continuous fibers in the tube must change their angle because the length of the tube will increase and the diameter will decrease. However, for a thin flat specimen, a fiber may be cut at the left and right sides of its gage section. Since the relative stiffness of the elastomer is many times lower than the fiber, the fiber will tend to stay at the same orientation and translate in the direction of deformation. If uncut fibers are placed in a “zigzag” or sinusoidal pattern, elongation of the article will produce more fiber rotation or re-orientation. Since fiber-reorientation is geometry specific, a simple model of a rubber muscle was created and will demonstrate the significance of fiber re-orientation.

The large deformations and significant reductions in cross-sectional area require selection of the appropriate definitions for stress and strain. The Lagrangian description considers properties relative to initial positions, while the Eulerian description considers properties relative to the current position. Because rubber is highly deformable, each definition has its advantages. Standard rubber models [10], such as the Mooney-Rivlin, Ogden, and Peng use the Cauchy (engineering) stress, $\sigma_i$, which is defined as
where $F_i$ is the applied force in the i direction and $A_o$ is the original cross-sectional area. The previously mentioned rubber models use extension ratio (stretch) instead of strain. Extension ratio, $a_i$ can be defined as

$$a_i = 1 + \varepsilon_i$$  \hspace{1cm} (2)$$

and

$$\varepsilon_i = \left( \frac{\Delta L}{L_o} \right)_i$$  \hspace{1cm} (3)$$

where $\varepsilon_i$ is engineering extensional strain in the i direction, $\Delta L$ is the change in length, and $L_o$ is the original gage length of the specimen. To maintain consistency with classical lamination theory, and to be able to use one of the above rubber material models, results are presented using the Lagrangian description (engineering stress and strain).

**Discussion of Linear Classical Lamination Theory**

Classical lamination theory use the orthotropic material properties $E_1$, $E_2$, $G_{12}$, and $\nu_{12}$ to describe the inplane axial modulus of elasticity, inplane transverse modulus of elasticity, inplane shear stiffness, and inplane major Poisson’s ratio, respectively, in each layer in a laminate.

For each layer:

$$\frac{-\nu_{12}}{E_1} = \frac{-\nu_{21}}{E_2}$$  \hspace{1cm} (4)$$

$$Q_{11} = \frac{E_1}{1 - \nu_{12} \nu_{21}}$$  \hspace{1cm} (5)$$

$$Q_{12} = \frac{\nu_{12} E_2}{1 - \nu_{12} \nu_{21}}$$  \hspace{1cm} (6)$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12} \nu_{21}}$$  \hspace{1cm} (7)$$

$$Q_{66} = G_{12}$$  \hspace{1cm} (8)$$

If the elastic constants $Q_{ij}$ are rotated we get the transformed stiffnesses $\overline{Q}_{ij}$:

$$\overline{Q}_{11} = Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta$$  \hspace{1cm} (9)$$
\[Q_{12} = (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta \cos^2\theta + Q_{12}(\cos^4\theta + \sin^4\theta) \]  
(10)

\[Q_{22} = Q_{11} \sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta \cos^2\theta + Q_{22} \cos^4\theta \]  
(11)

\[Q_{16} = (Q_{11} - Q_{22} - 2Q_{66})\sin\theta \cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta \cos\theta \]  
(12)

\[Q_{26} = (Q_{11} - Q_{22} - 2Q_{66})\sin^3\theta \cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta \cos^3\theta \]  
(13)

\[Q_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta \cos^2\theta + Q_{66}(\cos^4\theta + \sin^4\theta) \]  
(14)

Chou [8] suggests that the transformed stiffnesses \(\overline{Q}_{ij}\) can be approximated by:

\[\overline{Q}_{11} = E_2 + E_1 \cos^4\theta \]  
(15)

\[\overline{Q}_{12} = E_1 \sin^2\theta \cos^2\theta + \frac{E_2}{2} \]  
(16)

\[\overline{Q}_{22} = E_2 + E_1 \sin^4\theta \]  
(17)

\[\overline{Q}_{16} = E_1 \sin\theta \cos^3\theta \]  
(18)

\[\overline{Q}_{26} = E_1 \sin^3\theta \cos\theta \]  
(19)

\[\overline{Q}_{66} = E_1 \sin^2\theta \cos^2\theta + \frac{E_2}{4} \]  
(20)

and are suitable for flexible composites. When compared with Equations 9 to 14, Equations 15 through 20, did not predict shear stiffnesses properly at off-axis angles near 45°, were not used in the nonlinear model, and are not recommended unless the shear stiffness, \(G_{12}\), is not known.

By assuming the Kirchoff hypothesis that planes will remain planar when a plate is under bending, and treating \(u_o, v_o,\) and \(w_o\) as the mid-plane displacements of a laminated plate, the linear strains at a point \((x,y,z)\) are

\[\varepsilon_x = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} \]  
(21)

\[\varepsilon_y = \frac{\partial v_0}{\partial y} - z \frac{\partial^2 w_0}{\partial y^2} \]

\[\gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z \frac{\partial^2 w_0}{\partial x \partial y} \]

These strains can be put in the form:
Using classical lamination theory essentially all of the linear inplane and bending behavior of a laminate can be predicted. The theoretical development presented above is from reference [11]. Note that even though $E_{1}$, $E_{2}$, $G_{12}$, and $\nu_{12}$ were considered constants, nothing stops us from treating them as functions of strain to obtain correct stresses, forces or strains. When the elastic values listed above are not constant, we have the condition of material nonlinearity.

**Material Nonlinearity**

Classical lamination theory uses constants to describe the inplane axial modulus of elasticity, inplane transverse modulus of elasticity, inplane shear stiffness, and inplane major Poisson’s ratio, respectively, in each layer in a laminate. For fiber-reinforced elastomeric materials these terms may not be constant but can be functions of strain (we assume that viscoelastic characteristics need not be included in the nonlinear material model if quasi-static conditions are maintained).

The axial stiffness $E_{1}$ is highly dependent on axial fiber stiffness. If the reinforcing fiber stiffness is considered constant, the axial stiffness $E_{1}$ is considered constant as well. Inspection of the test results from [xx], from every material system at $0^\circ$ show linear stress-strain curves. This indicates that a single constant, $E_{1}$, can be used for the extensional stiffness of each material system. Values of $E_{1}$ for each material system can be viewed in the $0^\circ$ or first row of Table 3.5 (insert), which gives laminate longitudinal stiffness as a function of angle. FRE shear and transverse properties, however, are definitely not linear.

Predicting nonlinear material properties at high elongations is sometimes more of an art than a science. Typically, no one model can match all material properties. Chou [8] uses a third-order polynomial model. Clark [4] treats the transverse stiffness as a bi-linear curve. Since the material models are intended to represent the response of fiber-reinforced rubber, and the response of the FRE materials are similar to their rubber matrices, it’s likely that existing rubber models could be well suited to model the nonlinear transverse and shear stiffnesses. For this purpose, two rubber

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix}
= 
\begin{bmatrix}
\varepsilon_x^o \\
\varepsilon_y^o \\
\gamma_{xy}^o
\end{bmatrix}
+ z
\begin{bmatrix}
K_x \\
K_y \\
K_{xy}
\end{bmatrix}
\]
models are reviewed; the popular Mooney-Rivlin model, and a more accurate Ogden model, as well as Clark and Chou’s models.

The Bi-Linear Stress-Strain Model

Clark [4] assumes that the stress-strain curves for $E_1$, $E_2$ and $G_{12}$ are bi-linear, the theory is a good first approximation beyond linear stress-strain relations and is conceptually very easy to understand. Clark’s basic theory is depicted in Figure 2. Region I represents a lower moduli, usually associated with compression, while Region II is usually associated with tension. The value of strain $\varepsilon_1^*$, where the change occurs, is considered a material property. Although the bi-linear model does consider compressive behavior separately, it does not allow for multiple changes in stiffness.

![Figure 1 Bi-modular or bi-linear stress-strain material model.](image)

The Mooney-Rivlin Material Model

The two-coefficient Mooney-Rivlin [10] strain-energy function is the most widely used constitutive relationship in the stress analysis of elastomers. It is not the most accurate, however, if the material experiences both softening and stiffening during elongation. The model was derived by Mooney and Rivlin based on a linear relationship between stress and strain in simple shear. For incompressible materials the strain function can be expressed as

$$U = c_1(I_1 - 3) + c_2(I_2 - 3),$$  \hspace{1cm} (23)
where \( I_1 \) and \( I_2 \) are the principal invariants of the strain tensor. For a special case of uniaxial tension of an incompressible Mooney-Rivlin material, the stress-strain equation can be expressed as:

\[
S = 2(a - a^{-2})(c_1 + c_2a^{-1})
\]  

(24)

where \( S \) is the Cauchy or engineering stress (the ratio of force to original area) and \( a \) is the stretch or extension ratio \((1 + \varepsilon)\). For both the incompressible and compressible forms, the initial shear modulus is:

\[
G = 2(c_1 + c_2)
\]  

(25)

If the material is incompressible, the initial tensile modulus \( E \) is calculated by:

\[
E = 6(c_1 + c_2)
\]  

(26)

For a compressible Mooney-Rivlin material the initial modulus is:

\[
E = (9KG)/(3K+G)
\]  

(27)

where \( K \) is the initial bulk modulus [10].

The three-coefficient Mooney-Rivlin model [12] can be expressed as

\[
S = 2 \left( c_1a - \frac{c_2}{a^3} + c_3 \left( \frac{1}{a^3} - a \right) \right)
\]  

(28)

where \( c_1, c_2 \) and \( c_3 \) can be obtained through a curve-fitting algorithm.

**The Ogden Material Model**

The Ogden material model [10] relates the strain-energy density as a separable function of the principal stretches (extension ratios). For incompressible materials, the strain energy function can be expressed as,

\[
U = \sum_{i=1}^{3} \sum_{j=1}^{m} \frac{c_j}{b_j}(a_i^j - 1)
\]  

(29)

where \( c_j \) and \( b_j \) are the material coefficients and \( a_i \) are the three principal stretch ratios \((i=1 \text{ to } 3)\). If the coefficients \( c_j \) and \( b_j \) are chosen correctly, this material model can provide extremely accurate representations of the mechanical response of hyperelastic materials for large ranges of deformation.
Depending on the type of response needed, the coefficients $c_j$ and $b_j$ can be developed from a simple tensile stress-strain curve. For a simple tension test, the Ogden formulation can be expressed as,

$$\sigma = \sum_{j=1}^{n} c_j (a_j^{b_j} - 1 - a_j^{-1} (1 + 0.5b_j))$$

(30)

The number of coefficients needed to predict the stress-strain characteristics of an elastomer depends on the amount of accuracy desired by the user. Typically three sets of coefficients are sufficient to fit the data for most types of vulcanized rubbers exhibiting behavior close to the natural rubber. One methodology to calculate three sets of Ogden coefficients is given in reference [10].

![Figure 2](image_url)  
Comparison of several material models with silicone/cotton shear modulus.

A comparison of the third order polynomial, two-coefficient Mooney-Rivlin model, the three-coefficient Mooney-Rivlin model, and a six-coefficient Ogden model, as illustrated in Figure 3,
shows the relative accuracy of the four models. The polynomial relation worked well for some materials but not others. The Ogden model, while slightly more computationally intensive, is the most accurate.

**Implementation of the Ogden Material Model**

Implementation of the Ogden model consists of two main steps: 1) making sure the experimental data is in the correct form, and 2) obtaining the six coefficients for each property.

Using the relations explained in Reference xxx? (Equations 27 - 31), shear and transverse moduli were obtained as a function of strain. We must remember, however, that when stress and strain were measured, the linear definition of strain was used:

\[ \varepsilon_i = \left( \frac{\Delta L}{L_0} \right)_i \]

and that:

\[ \varepsilon_x = \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_0}{\partial x^2} \]

is the definition of linear strain in the axial direction. Since the specimens saw extremely high elongations, the stress resulting from the strain included the contribution of geometric nonlinearity. In the axial direction the nonlinear definition of axial strain is:

\[ e_x = \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial u_0}{\partial x} \right)^2 - z \frac{\partial^2 w_0}{\partial x^2} \]

Because there was no bending in the specimens tested, the bending contributions in Equations 40 and 41 can be ignored. The contributions of geometric nonlinearity to the stress-strain curve, and hence stiffness, can not be ignored and must be removed in order to have accurate nonlinear material properties.

The linear strain \( \varepsilon_x \) was plotted as a function of \( e_x \). A curve fitting program was used to find an extremely accurate relation that expresses \( \varepsilon_x \) as a function of \( e_x \):

\[ \varepsilon_x = -0.01292 e_x^4 + 0.097 e_x^3 + 0.3154 e_x^2 + 0.96428 e_x \]

The reduced strain was used to obtain a reduced stress by the following relation:

\[ \sigma_i = \sigma_{i-1} + (\varepsilon_i - \varepsilon_{i-1}) E_{inst} \]

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where $E_{\text{inst}}$ is the instantaneous Young’s modulus, and $i$ represents the current strain state. The reduced stress is plotted as a function of the measured linear strain $\varepsilon_x$, and the nonlinear shear and transverse properties are obtained from the paired values.

Now that the nonlinear properties have been put in the proper form, and geometric nonlinearity has been removed, the six Ogden coefficients for each material property must be obtained. The Ogden model produces stress as a function of extension ratio, and since the extension ratio is a constant plus strain, the Ogden model can be considered as predicting stress as a function of strain. The instantaneous derivative of the predicted Ogden stress with respect to extension ratio yields:

$$[G_{12}, E_2] = \sum_j c_j \left[ (b_j - 1)a^{b_j - 2} + (1 + 0.5 b_j)a^{-(2 + 0.5 b_j)} \right], \quad (36)$$

which is the instantaneous shear or transverse stiffness as a function of strain. The above relation, with three coefficients for $c_j$ and three for $b_j$ provides a much cleaner and more accurate form for the nonlinear stiffnesses, than the original Ogden model. A curve fitting program, Sigma Plot, was used to obtain the six coefficients for each material property.

The stiffness versus strain values were smoothed where local irregularities occurred. Also, some properties showed a small “knee” in the data at very small strains. Since, at very small strains, variations in stiffness would contribute little to overall stress, these “knees” were removed. The resulting coefficients are given in Table 1. Because the average elongation at 45° is approximately 40%, while elongations at 90° can exceed 200%, an axial failure strain, $\varepsilon_{xg}$, for the 45° specimens is presented as well. The failure strain is discussed in the section on solution procedures.

The Ogden coefficients for the transverse strain are given in Table 2. Since the Ogden model was not intended to predict the stress-strain response of the four material combinations beyond the failure strains of the 90° specimens, no failure strain is reported, or needed in the computer model. A visual depiction of how well the Ogden rubber model fits the nonlinear shear and transverse properties is shown in Figures 4 and 5, respectively. Notice that there is very good correlation with both shear and transverse stiffnesses. There is a slight divergence at the highest strains for the silicone/fiberglass shear stiffness. This divergence shows up in some of the silicone/fiberglass predictions made later.
Figure 3  Experimental and modeled nonlinear shear stiffness for each material system with a [+45/-45]_2 layup.

Figure 4  Experimental and modeled nonlinear transverse stiffness for each material system with a [+90/-90]_2 layup.
For silicone/cotton shear stiffness, the Ogden model is started at approximately 15% strain in order to optimize the material properties at higher deflections, although it still models the silicone/cotton shear property adequately at lower strains. Although the Ogden model works very well, and better than other models considered, like any curve fit, care must be taken to ensure the fit is best in the regions of most interest.

**Geometric Nonlinearity**

The limitation of classical lamination theory that strain is assumed to be a linear function of displacement and that strains are small is now addressed. Although some very rigid elastomers...
may behave in a linear manner, most elastomers can strain from 25% to 800%. Linear strain-displacement theory assumes that:

\[ \varepsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_0}{\partial x^2} \]
\[ \varepsilon_y = \frac{\partial v_0}{\partial y} - z\frac{\partial^2 w_0}{\partial y^2} \]
\[ \gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w_0}{\partial x \partial y} \]

The geometrically nonlinear strain-displacement theory, also known as the Lagrangian nonlinear strain tensor, is

\[ e_x = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\left(\frac{\partial u_0}{\partial x}\right)^2 + \left(\frac{\partial v_0}{\partial y}\right)^2 + \left(\frac{\partial w_0}{\partial z}\right)^2\right) - z\frac{\partial^2 w_0}{\partial x^2} \]
\[ e_y = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\left(\frac{\partial u_0}{\partial x}\right)^2 + \left(\frac{\partial v_0}{\partial y}\right)^2 + \left(\frac{\partial w_0}{\partial z}\right)^2\right) - z\frac{\partial^2 w_0}{\partial y^2} \]
\[ \Gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial y} \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w_0}{\partial x \partial y} \]

Because we are concerned primarily with inplane deformation, with no bending (although bending is considered in Equations 46 and 47), the definitions for strain can be simplified. The following relations assume that off-axis strains are fairly small, rotations about the x and y axes are moderately small, and that rotations about the z axis are negligible. Accordingly, the strain-displacement relations are:

\[ \varepsilon_x = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\left(\frac{\partial u_0}{\partial x}\right)^2 \right) - z\frac{\partial^2 w_0}{\partial x^2} \]
\[ \varepsilon_y = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\left(\frac{\partial v_0}{\partial y}\right)^2 \right) - z\frac{\partial^2 w_0}{\partial y^2} \]
\[ \gamma_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} - 2z\frac{\partial^2 w_0}{\partial x \partial y} \]
where $e_x$, $e_y$, and $\gamma_{xy}$ represent the axial, transverse and shear strains that are used in the modified nonlinear model. Note that the definition for shear strain remains the same. This is consistent with formulations that Chou [8] has used.

**Implementation of Nonlinear Model**

The nonlinear additions to classical lamination theory were coded into an existing composites analysis program. The initial DOS-based program, called PCLAM, was written by Dr. Steven L. Folkman and Larry Peel, while at Utah State University. The program is written in Lahey Fortran, which has additional graphics and text capability, and allows the easy creation of interactive DOS-based programs.

**The Computer Code**

The modified program, PCFRE3, was compiled using the Lahey LF90 Fortran compiler. The editor, compiler, and compiled program operate well under Windows 3.11 and Windows 95. A series of figures (screen captures) showing the flow and output of the program are presented in Appendix A. PCFRE3 code added for the current research is included in Appendix B. A linear material properties data file (MDAT2.DAT), a nonlinear material properties data file (FRE-DAT.DAT), and a nonlinear stress-vs.-strain output file (FREOUT.DAT) that uses input from Appendix A, are included in Appendix C.

**Method of Solution**

To analyze a fiber-reinforced elastic laminate, the user must have initial linear material properties, the six Ogden coefficients for the shear and transverse stiffness, the shear failure strain, and a lay-up definition. After the material properties are entered into PCFRE3, they are stored in the material data files and do not need to be re-entered. After the angle-ply layup is defined, a laminate ABD stiffness matrix is determined using the initial linear properties. From the laminate stiffness matrix a laminate Poisson’s ratio is calculated. The user moves to the nonlinear menu and keys in the specimen width, number of calculation steps, initial length, and the increment of length that the specimen will be elongated at each calculation step.

The incremental axial linear and nonlinear strains, and the total linear and nonlinear axial strains respectively, are defined by:
\[ \Delta \varepsilon_x = \frac{\Delta L}{L} \]  
\[ \Delta e_x = \frac{\Delta L}{L} + \frac{(\Delta L/L)^2}{2} \]  
\[ \varepsilon_x = n \frac{(\Delta L)}{L} \]  
\[ e_x = n \frac{(\Delta L)}{L} + \frac{(n\Delta L/L)^2}{2} \]

where \( \Delta L \) is the length increment, \( L \) is the initial length, and \( n \) is the number of increments calculated to that point. The transverse linear strain is calculated by the relation:

\[ \varepsilon_y = -v_{xy}\varepsilon_x \]  

where \( v_{xy} \) is the laminate Poisson’s ratio. The same procedure is used in calculating all other incremental and total transverse strains.

Because geometric nonlinearity has been removed from the material properties, linear strain is used to determine the instantaneous shear and transverse moduli. At the \( n \)th iteration the total linear strain is averaged with the \( (n-1) \)th linear strain. The resulting strain is used to obtain new stiffness values for \( E_2 \) and \( G_{12} \), which are then used to recalculate the laminate stiffness (ABD) matrix. If the \( n \)th total linear strain was used to obtain the laminate stiffness matrix, the total stress is slightly over-predicted. If the strain exceeds \( \varepsilon_{xg} \) for the material being used, the program calculates a shear stiffness based on a line tangent to the shear-strain curve at \( \varepsilon_{xg} \). If the predicted shear modulus is negative, a small positive value for shear stiffness is used.

The total axial engineering stress at the \( n \)th iteration is predicted by:

\[ (\sigma_x)_n = (\sigma_x)_{n-1} + (rA_{11}((e_x)_n - (e_x)_{n-1}) + rA_{12}((e_y)_n - (e_y)_{n-1}))/w \]  

where \( rA_{12} \) and \( rA_{12} \) are the recalculated inplane axial and coupling stiffness, and \( w \) is the laminate or specimen width. Since the experimental data was recorded as linear strain vs. total stress, the program outputs to the screen and to an output file, at the \( n \)th iteration, total linear strain and total stress. If bending of the laminate was expected, bending contributions could be added in a similar manner.
Laminate Poisson’s ratios are also predicted by the nonlinear model. The model predicted that the fiberglass-reinforced silicone and urethane specimens would exhibit extremely high Poisson’s ratios of 5 to 32 from fiber angles of approximately 1° to 25°. The predicted Poisson’s ratios are displayed in Figure 6. The high Poisson’s ratios provide another reason why gripping problems are noted with the low off-axis fiberglass-reinforced specimens. Poisson’s ratios are functions of fiber angle, fiber stiffness, and elastomer stiffness. Further measurements of Poisson’s ratios from FRE specimens should be conducted. The ability to tailor FRE laminates with extremely high Poisson’s ratios, and perhaps high negative Poisson’s ratios open the door for new applications such as a fastener that expands transversely when extended, or unique actuators.

![Figure 5 Predicted Poisson’s ratios as a function of angle for each material system.](image)

Discussions of general stress-strain characteristics predicted by the improved nonlinear model are presented in Chapter 5 along with comparisons to measured stress-strain responses of the four fiber-reinforced elastomeric material systems.
PREDICTED AND EXPERIMENTAL STRESS-STRAIN RESPONSES

Predictions using PCFRE3 were compared with the experimental data from Reference [xx]. The predicted and experimental results at 0° for the fiberglass-reinforced elastomeric specimens do not match because the test fixture did not completely load the 0° fiberglass-reinforced specimens.

Predicted results are compared with average experimental results for each specimen type. The cotton-reinforced silicone rubber stress-strain results are considered first.

Cotton-Reinforced Silicone

Silicone rubber and cotton fibers represent the most compliant matrix and fiber components of the four combinations investigated. Predicted and average stress-strain results from the 0° to 45° balanced angle-ply specimens are shown in Figure 5.1. Specimens with fibers at 0° are the stiffest and are the left-most curve. Predicted and average stress-strain results from the 45° to 90° specimens are shown in Figure 5.2 (specimen results at 45° are repeated here to facilitate comparison between the two related graphs). Correlation between predicted and measured results are good except for 15°. The predicted and measured results at 15° have approximately the same slope. At 30° and 45° the predicted results follow the same trends and stiffnesses, but are slightly low. This is expected since the initial “knee” in the cotton/silicone shear stiffness was not modeled by the Ogden relation. Up to approximately 50% strain, stress-strain curves for 60° to 90° specimens are very similar. This has ramifications for elastic tailoring, since at 60°, a laminate has considerably more shear and transverse stiffness.

Fiberglass-Reinforced Silicone

The fiberglass-reinforced silicone composite specimens combine a stiff fiber and a compliant matrix. The predicted and average results from the 0° to 45° off-axis angles are shown in Figure 5.3. The predicted and average results from the 45° to 90° off-axis angles are shown in Figure 5.4. Correlation is excellent except at strains higher than 25% for 60° and 75°, and at 0° as noted earlier. Looking back at the Ogden model of the silicone/glass shear strain, the model diverged slightly at the shear failure strain which is approximately 25%. This divergence caused the 60° and 75° results to be over-predicted at higher strains. The results at 90° are modeled better because shear has little effect at the high angle.
Figure 6 Predicted and measured cotton/silicone stress-strain behavior from $[\pm 0^\circ]_2$ to $[\pm 45^\circ]_2$.

Figure 7 Predicted and measured cotton/silicone stress-strain behavior from $[\pm 45^\circ]_2$ to $[\pm 90^\circ]_2$. 
Figure 8  Predicted and measured fiberglass/silicone stress-strain behavior from [± 0°]_2 to [± 45°]_2.

Figure 9  Predicted and measured fiberglass/silicone stress-strain behavior from [± 45°]_2 to [± 90°]_2.
Cotton-Reinforced Urethane

Predicted and average stress-strain results from urethane/cotton specimens with 0° to 45° off-axis angles are shown in Figure 5.5. As the angle is increased, specimen stiffness and strength decrease. Figure 5.6 shows predicted and average test results for 45° through 90° specimens. The softening effect of the urethane rubber is readily apparent for 60° and greater angle-ply specimens. Similar to the silicone/cotton results, at strains less than 25%, there is little difference between the 75°, the 90°, and to a lesser extent, the 60° stress-strain curves, which PCFRE3 accurately predicts. From 0° to 30°, and 75° to 90° the predictions and average results compare very favorably. At 45° and 60° the predictions are somewhat low, but show the same trends. The reasons for this are related to the laminate Poisson’s ratio used to calculate $G_{12}$, as discussed later.

![Figure 10](image-url)  
**Figure 10** Predicted and measured urethane/cotton stress-strain behavior from $[\pm 0^{\circ}]_2$ to $[\pm 45^{\circ}]_2$.

Fiberglass-Reinforced Urethane

Predicted and average stress-strain results for urethane/fiberglass specimens at 0° to 45° off-axis angles are shown in Figure 5.7. As noted earlier, the 0° experimental and predicted results do not correspond. Predicted and experimental results for 45° to 90° specimens are shown in Figure 5.8. Due to manufacturing error, the “30°” specimens were actually 37°, and the “60°” specimens
Correlation between predicted and experimental results are quite good except at 37° and 53°. It’s quite likely that the test data is faulty for these specimens, since correlation is very good at other angles.

**DISCUSSION OF PREDICTED RESULTS**

Correlation between experimental and predicted results at low and high off-axis angles were typically very good. This indicates that the model predicts fiber- and matrix-dominated stress-strain response very well. The predicted results at 45° and 60° followed the same trends as experimental results, and except for the silicone/glass predictions (which were very close) were slightly low. This would indicate that the shear-dominated response of the specimens is not being modeled sufficiently. The first attempt to rectify this apparent concern was to use Chou’s approximations (Equations 4.17 - 4.22) for transformed layer stiffnesses. Unfortunately, correlations with these approximations produced worse results than those presented. Other modified forms of the transformed layer stiffnesses were developed and considered. These also produced inconsistent results.
Figure 12  Predicted and measured urethane/glass stress-strain behavior from $[\pm 0^\circ]$ to $[\pm 45^\circ]$.

Figure 13  Predicted and measured urethane/glass stress-strain behavior from $[\pm 45^\circ]$ to $[\pm 90^\circ]$. 
Laminate Poisson’s ratios at 45° were predicted for each material system, using initial properties and classical lamination theory, and compared with measured values. They are given in Table 5.1. Classical lamination theory always predicts a 45° laminate Poisson’s ratio less than those measured. In FRE material systems such as those currently considered where the fiber stiffness is several orders of magnitude higher than the matrix stiffness, the Poisson’s ratio is predicted to be approximately 1, but never greater. Since excellent correlation was shown for the silicone/fiberglass material system, which used a calculated Poisson’s ratio, one of several conclusions can be made: 1) the laminate Poisson’s ratios need to be re-measured under more tightly controlled conditions; 2) there are additional mechanisms affecting the response of shear-dominated FRE specimens, which are not currently included in the nonlinear model; or 3) the Poisson’s ratio is not constant and must be measured in a manner similar to the other nonlinear properties. It is highly likely that all three factors contribute to the discrepancy, and should be explored in future work. For the present a better correlation can be obtained by using measured nonlinear shear stiffnesses that employ a calculated 45° laminate Poisson’s ratio.

The nonlinear trends of the experimental results are predicted quite well by the current model, even in shear dominated regions. Inconsistent correlations at a few instances show no trend and are related to Poisson’s ratio and fabrication issues rather than modeling concerns.

### TABLE 3  Measured and predicted Poisson’s ratios ($\nu_{xy}$) for each material system

<table>
<thead>
<tr>
<th>Material System</th>
<th>Predicted Poisson’s ratios, ($\nu_{xy}$)</th>
<th>Measured Poisson’s ratios, ($\nu_{xy}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cotton/Urethane</td>
<td>0.955</td>
<td>1.29</td>
</tr>
<tr>
<td>Cotton/Silicone</td>
<td>0.982</td>
<td>1.42</td>
</tr>
<tr>
<td>Fiberglass/Urethane</td>
<td>0.997</td>
<td>1.14</td>
</tr>
<tr>
<td>Fiberglass/Silicone</td>
<td>0.998</td>
<td>0.998*</td>
</tr>
</tbody>
</table>

* Calculated, based on $\nu_{xy}=3.03$ @ 30° measurement.
FIBER RE-ORIENTATION AND THE “RUBBER MUSCLE”

A “rubber muscle” is modeled as a composite cylinder with internal pressure. Basic formulation for the composite cylinder was obtained from Whitney [13], with modifications made for material nonlinearity and fiber re-orientation. The model uses the nonlinear material model of PCFRE3. The rubber muscle is fabricated using the same balanced angle-ply lay-up scheme as the experimental FRE specimens. Because expected strains are less than 10%, and for simplicity, nonlinear strain-displacement relations are not included in the rubber muscle model. The length and diameter of the tube are updated after each iteration, however, to provide a measure of nonlinearity. Fiber angle re-orientation is included as a function of geometry (length and diameter) changes as discussed below.

The axi-symmetric composite cylinder with an angle-ply lay-up is considered to initially have a constant radius $R$ and length $l$. Because of symmetry along the longitudinal axis, the cylinder experiences only axial displacement $u$, radial deflection $w$. If the cylinder is restrained at its ends, an axial stress-resultant $N_x$ is produced instead of axial displacement.

The governing equations for such a composite cylinder are:

\begin{align}
A_{11} \frac{\partial^2 u}{\partial x^2} + \left( \frac{A_{12}}{R} \right) \frac{\partial w}{\partial x} &= 0 \tag{46} \\
A_{66} \frac{\partial^2 v}{\partial x^2} - B_{16} \frac{\partial^3 w}{\partial x^3} &= 0 \tag{47}
\end{align}

and

\begin{align}
\frac{A_{12}}{R} \left( \frac{\partial u}{\partial x} \right) - B_{16} \left( \frac{\partial^3 v}{\partial x^3} \right) + D_{11} \left( \frac{\partial^4 w}{\partial x^4} \right) = p + N_x^i \left( \frac{\partial^2 w}{\partial x^2} \right) \tag{48}
\end{align}

where $x$ represents the axial direction, and the laminate stiffnesses are as defined earlier. The laminate stiffnesses are updated after each iteration. The initial stress-resultant $N_x^i$ changes as the internal pressure $p_o$ changes, and is defined as:
Because the cylinder is axi-symmetric the transverse or circumferential displacement \( v \) is identically zero. Therefore Equation 55 can be discarded, as well as the second term of Equation 56. The circumferential strain, is not zero, however, but is a function of the out-of-plane displacement:

\[
\varepsilon_s = \frac{w}{R} \tag{50}
\]

where \( s \) represents the circumferential direction.

At the cylinder end \( x = 0 \), the boundary conditions are:

\[
w = 0 \tag{51}
\]

\[
\frac{\partial w}{\partial x} = 0 \tag{52}
\]

\[
N_x = 0 \left( \frac{\partial u}{\partial x} = 0 \right), \text{ or } u = 0 \tag{53}
\]

At the cylinder end \( x = l \), the boundary conditions are:

\[
w = 0 \tag{54}
\]

\[
\frac{\partial w}{\partial x} = 0 \tag{55}
\]

\[
N_x = 0 \left( \frac{\partial u}{\partial x} = 0 \right), \text{ or } u = 0 \tag{56}
\]

The “rubber muscle” actuator model is set up so that both initial axial force and axial displacement, \( u \), are calculated as a function of pressure, but the force calculation assumes that axial displacement is zero, and the displacement calculation assumes that axial force is zero. To formulate the axial displacement correctly, however, an additional boundary condition was used for the axial displacement, at \( x = l/2 \):

\[
u = 0 \tag{57}
\]

Because of axi-symmetry, all relations are functions of only the axial direction \( x \). The partial differential equations become ordinary differential equations and can be solved by direct integration.
Using the boundary conditions, a relation for the out-of-plane displacement \( w \), was obtained, and used to obtain the axial displacement \( u \). Hyperbolic and polynomial expressions were used to give a shape for the displacement \( w \). Since both predicted shapes were very similar, the polynomial relation was used for simplicity. Assuming a 4th-order polynomial, the out-of-plane displacement can be represented as:

\[
w(x) = F\left(l^2x^2 - 2lx^3 + x^4\right)
\]  

(58)

where the coefficient \( F \) is an unknown coefficient. Substituting Equation 66 into Equation 54, and using the boundary conditions of 61 or 64, and 65, the axial displacement is:

\[
u(x) = \frac{A_{12}}{A_{11}R}\left(F\left(l^2x^3 - \frac{lx^4}{5} + x^5 - \frac{l^5}{60}\right)\right)
\]  

(59)

The coefficient \( F \) is obtained by substituting Equations 66 and 67 into Equation 56, where \( p_o \) is the internal pressure:

\[
F = \frac{16p_o}{\pi^2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{nm} \sin \frac{m\pi x}{l}
\]  

(60)

\[
= \frac{A_{12}^2}{A_{11}^2R^2}\left(l^2x^2 - 2lx^3 + x^4\right) + \frac{p_oR}{2}\left(2l^2 - 12lx + 12x^2\right) - 24D_{11}
\]

The summations are used to represent the constant internal pressure. Knowing the displacements, the equivalent initial axial force can be determined:

\[
F_x = (2\pi R)N_x = 2\pi RA_{11}\left(\frac{u}{l}\right) + 2\pi RA_{12}\left(\frac{w}{R}\right)
\]  

(61)

This formulation assumes that the axial force is due to the radial expansion of the muscle minus the axial contractive effects. The formulation is consistent with experimental observations. Another way to understand the mechanism is to think of two people pulling tightly on the ends of a rope. If someone else pulls transversely on the middle of the rope, the two end people will be drawn together with considerable force.

After axial and radial expansions are determined, and geometry changes are calculated, fiber re-orientation can be considered. Using the instantaneous fiber angle, an effective muscle length is calculated for each unique ply:
At each iteration, the incremental pressure is used to calculate an incremental change in radius and length, therefore, the new fiber angle for each ply, at each step or iteration is:

\[
\theta_i = \tan^{-1}\left(\frac{2\pi R + 2w}{l_{\text{eff}} + l_{\text{eff}}\left(\frac{u}{t}\right)}\right)
\]

At each iteration, the positive transverse strain is used to update the material properties, using the capabilities of the nonlinear material model discussed earlier. The model calculates the total cylinder contraction, and the average diameter change.

At a given pressure, the contractive force will likely decrease significantly as the muscle contracts. That characteristic will be incorporated into a later version of the rubber muscle model. The current model only predicts initial contractive force. As the fiber angle of an angle-ply FRE cylinder increases, the amount of cylinder contraction decreases. After approximately \(55^\circ\), the cylinder begins to elongate. The current rubber muscle model does not accurately predict the force developed under such axial expansion, hence fiber angles should be kept lower than \(55^\circ\).

The rubber muscle model was added to the code of PCFRE3, the complete nonlinear fiber-reinforced elastomer and rubber muscle model is called PCFRE4. The rubber muscle model is implemented as another interactive menu, similar to the nonlinear fiber-reinforced elastomer menu.

Photographs of a fabricated rubber muscle, and predictions from the rubber muscle model, using the nonlinear properties obtained from Reference [xx], are presented and discussed below. The discussion shows how contractive force and fiber re-orientation vary, based on material type, pressure, muscle length, and muscle diameter.

**Predictions from the “Rubber Muscle” Model**

An inflated “rubber muscle” that was fabricated as part of the current work is shown in Figure 5.9. When inflated with air, the rubber muscle actuator exhibits considerable initial contractive force. The fabricated rubber muscle use small Kevlar™ tows oriented at a \(\pm 20^\circ\) lay-up in a urethane matrix. A simple model of the rubber muscle was developed
and is presented in Chapter 4. The rubber muscle model is an addition to, and uses the nonlinear fiber-reinforced elastomer model discussed in Chapter 4. The rubber muscle model is still somewhat rudimentary, and is intended to provide qualitative answers, however the magnitudes of the initial forces, angle re-orientation, and displacements are consistent with the response of the fabricated muscle. Direct comparison of the contractive response of the fabricated rubber muscle with predicted results has proven difficult because material properties for the fabricated rubber muscle have not been obtained and the extremely high Poisson’s ratios of the inflated muscle cause the metal fittings on the ends of the muscle to pull out when clamped in a test fixture.

Using an angle-ply lay-up, $[\pm \theta]_2$, with an initial fiber angle of $15^\circ$, a ply thickness of 0.635 mm (0.025 in), an initial diameter of 12.7 mm (0.5 in), and an initial muscle (actuator) length of 254 mm (10 in), results are presented for the four sets of material properties obtained in Chapter 3. Figure 5.10 illustrates initial contractive force versus pressure for each material system. As shown in Figure 5.10, initial contractive force is very independent of material type, for the same pressures and geometry.

Fiber angle re-orientation as a function of pressure for each material system is shown in Figure 5.11. The fiber angle changes are very much a function of material properties. The greatest fiber angle changes are evident where axial stiffness is lower. The tendency
of the muscle diameter to increase, and the muscle length to decrease, since they are also measures of geometry, follow the same trends as fiber angle re-orientation.

Varying the rubber muscle wall thickness has essentially no effect on initial contractive force, but did affect geometry changes such as length, diameter, and fiber angle. Increasing muscle length and/or muscle diameter increases contractive force and fiber re-orientation, because of the effective increase in surface area. Evidence of nonlinearity is more prominent for longer muscle lengths, larger diameters and lower initial fiber angles.

Changing initial fiber angle has a very large effect on initial contractive force, as shown in Figure 5.12, and on the amount of fiber re-orientation, shown in Figure 5.13. Changing initial fiber angle is essentially the same as changing axial stiffness so the results shown in Figure 5.13 are consistent with Figure 5.11. The extremely large fiber angle re-orientation for the lower initial fiber angles is possible because the lower transverse stiffnesses at lower fiber angles cause the muscle to “bulge out” more.
To understand the predicted results better, let us go back to the analogy of two people pulling on the ends of a rope, as discussed in Chapter 4. Assuming the rope is strong enough that it won’t break, if one were to pull transversely on the rope, the amount of axial contractive force induced is not a strong function of what the rope is made of, or its diameter. If one is to increase the length of the rope, however, the same transverse force will produce a higher axial force, but decrease axial displacement. If the rope is somewhat compliant, it’s possible to see how the same force would be transmitted as in the “stiff” rope, but transverse deflections would be greater. If the rubber muscle is used as an actuator, the capabilities of an improved rubber muscle model will be useful in tailoring the force and deflection response.

Figure 16  Predicted fiber angle change as a function of pressure for each material system with a $[\pm 15^\circ]_2$ lay-up.
Figure 17  Predicted initial contractive force as a function of pressure for different initial fiber angles.

Figure 18  Predicted fiber angle change as a function of pressure for different initial fiber angles.
SUMMARY OF THE NONLINEAR MODEL

An improved model for the mechanical response of fiber-reinforced elastomeric composites has been presented. The model uses classical lamination theory as its basis. Material nonlinearity is included in the form of nonlinear orthotropic material properties that are functions of linear extensional strain. Nonlinear shear and transverse properties were obtained from the experimental results of fiber-reinforced elastomeric (FRE) angle-ply specimens at 0°, 45°, and 90° as discussed in Reference [xx]. Axial stiffness and Poisson’s ratio are considered constant. Geometric nonlinearity is removed from the transverse and shear stiffnesses, in order that each nonlinear contribution might be considered separately. A bi-linear model, a two-coefficient Mooney-Rivlin model, a three-coefficient Mooney-Rivlin model and a six-coefficient Ogden model were considered in attempts to model the nonlinear material properties. The highly accurate Ogden model was chosen to represent the nonlinear stiffening and softening stiffnesses. Geometric nonlinearity is included through the addition of the nonlinear extensional terms of the Lagrangian strain tensor. The nonlinear strain-displacement relations and the nonlinear material models were added to the code of a pre-existing composites analysis software package. The improved analytical tool will aid in understanding the behavior of FRE materials and will enable their use in stiffness- and deformation-tailored smart structures.

Stress-strain predictions from the nonlinear laminated plate model are compared with the measured stress-strain response of balanced angle-ply specimens with off-axis angles ranging from 0° to 90° in 15° increments. Correlation between predicted and test results range from fair to excellent. Fiber and matrix dominated stress-strain responses are modeled very well. The model also gives fair to good comparisons for shear dominated modes, but more investigation needs to be made into laminate Poisson’s ratios. There are also a few instances where the test data appears to be flawed, hence correlation was inconclusive. Comparison of predicted with mechanical responses provides additional insight into FRE response mechanisms, since one is typically able to vary parameters in a model that are not easily reproduced by experiment.

A model of a “rubber muscle” actuator is presented. The model consists of a composite cylinder that includes the nonlinear material model and fiber rotation (fiber re-orienta-
A model of a “rubber muscle” actuator is presented. The model consists of a composite cylinder that includes the nonlinear material model and fiber rotation (fiber re-orientation). The model is useful in aiding the qualitative understanding of the mechanical behavior of inflated FRE cylinders or “rubber muscles” actuators and fiber re-orientation. Predictions from the “rubber muscle” actuator model reveal significant insights about the mechanical behavior of FRE muscles or flexible composite cylinders. The initial contractive force is essentially independent of material type, but very dependent on pressure, initial fiber angle, diameter, and length. Fiber re-orientation and contraction are functions of geometry, material type, and pressure. Contractive force decreases as the “muscle” contracts. Initial contractive forces are approximately 120 times the force that a hydraulic cylinder with the same internal pressure and diameter would produce. A refined rubber muscle model would be useful in the design of rubber muscles, and will aid their use as actuators in flexible and smart structures. The rubber muscle shows promise as an embedded or nonlinear actuator.

VIII. ACKNOWLEDGEMENTS

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REFERENCES